The Length and Mass Scales of Cosmology and Astrophysics from Quantum Scales

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The length and mass scales of cosmology and astrophysics are shown to derive from quantum scales. The Quantum/Classical connection maps the Bohr radius, the reduced Compton wavelength and the classical electron radius – three 'quantum' length scales that lie in geometric progression with common ratio α, the fine structure constant – onto three 'classical' length scales: the radius of the observable universe; the radius of the stellar halo of the geometric-mean-sized galaxy that is typified by the Milky Way; and the radius of the planetary system of the geometric-mean-sized star that is typified by the sun. By way of the Quantum/Classical connection, the three classical length scales lie in geometric progression with common ratio equal to $\alpha^{5/2}$ (=4.55x10^{-6}). This observation has opened up a framework of multiplicative relationships between cosmological and astrophysical scales based on powers of $\alpha^{5/2}$. The astrophysical scales of the model refer to the geometric mean values of distributions of scales. 'Local' scales take very nearly ideal (geometric mean) values. The baryonic mass of the Milky Way and the mass of the sun approximate very closely to the baryonic mass of the Hubble sphere multiplied by α^5 and α^{10}, respectively. The total mass of the Milky Way, the mass of the supermassive black hole Sgr A* at the centre of the Milky Way and the geometric mean mass of black hole-black hole gravitational wave event progenitors approximate very closely to the total mass/energy of the Hubble sphere multiplied by α^5, $\alpha^{15/2}$ and α^{10}, respectively. The results indicate that the total mass/energy of the Hubble sphere increases with time and that black holes grow at Hubble rate. The Schwarzschild radius of the 'ideal' star equals the radius of the star multiplied by $\alpha^{5/2}$. By way of the Quantum/Classical connection, the radius and mass of the ideal star map onto the mass and atomic radius, respectively, of the stable nuclide ^{56}Fe. The nearly-ideal sun is the counterpart of the stable nuclide ^{53}Cr.

1 Introduction

The Quantum/Classical connection [1, 2] is applied to a trio of length scales: the Bohr radius; the reduced Compton wavelength; and the classical electron radius, and is found to map the 'quantum' length scales onto, respectively: the radius of the observable universe; the radius of the stellar halo of the geometric-mean-sized galaxy that is typified by the Milky Way; and the radius of gravitational influence on baryonic matter of the geometric-mean-sized star that is typified by the sun. While the three 'quantum' scales lie in geometric sequence with common ratio α = 1/137.036 (the fine structure constant), the three corresponding 'classical' scales lie in geometric sequence with common ratio $\alpha^{5/2}$ = 4.55 x 10^{-6}. The skeleton of multiplicative relationships disclosed by the Quantum/Classical connection is fleshed out: first, the relationships between length scales and, second, the relationships between mass scales. Black holes and stars are then given additional attention.

Planck units (ħ = c = G = 1) are used throughout the paper, which explains the apparently unbalanced dimensions in equations. The values of Planck length, Planck Mass and Bohr radius used in the calculations are 2018 CODATA recommended values. All mentions of mean value refer to the geometric mean of a distribution of sizes.

2 The Quantum/Classical Connection

The Quantum/Classical connection relates length and mass scales in $\mathrm{AdS}_5\times \mathrm{S}^5$ spacetime with scales in four-dimensional spacetime. Using Planck units, the Q/C connection between length scales is written as

$$2{r_\mathrm{Q}}^5 = {R_\mathrm{C}}^2 \tag{1}$$

where r_Q is a 'quantum' length scale and R_C is a corresponding 'classical' length (radius) scale. Volumes in $\mathrm{AdS}^5\times \mathrm{S}^5$ spacetime correspond to areas in four-dimensional spacetime. The mass/energy density of space is scaled down from Planck scale by a factor $2(a_\mathrm{o}/l_\mathrm{Planck})^5$ = 1/(1.33x10^{-123}) when measured in the quantum realm [1] and a corresponding factor $(R_\mathrm{OU}/l_\mathrm{Planck})^2$, of equal size, when measured in the classical realm [2]. The Q/C connection maps the Bohr radius a_0 onto a length scale of 14.37 Gpc. The radius R_OU of the observable universe is ≈14.3 Gpc [3]. Our result $2{a_0}^5 = {R_\mathrm{OU}}^2$ tells us that the radius of the observable universe is unchanging.

3 Length Scales

The reduced Compton wavelength $\lambda_\mathrm{e}/2\pi$ of the electron and the classical electron radius r_e are the length scales at which quantum field effects and then renormalisation become important in quantum electrodynamics, the quantum theory of light that describes the interaction of electromagnetic radiation and matter. Using Planck units, $\lambda_\mathrm{e}/2\pi = 1/m_\mathrm{e}$ and $r_\mathrm{e} = \alpha/m_\mathrm{e}$, where m_e is the mass of the electron and α is the fine structure constant. Since the Bohr radius, $a_0 = 1/\alpha m_\mathrm{e}$, corresponds by way of the Q/C connection to the radius of the observable universe, the classical length scales corresponding to $\lambda_\mathrm{e}/2\pi$ and r_e were calculated using (1) and found to be equal to 65.0 kpc (212,000 light years) and 61,000 AU (0.964 lyr), respectively. The first of these length scales seems to refer to the radius of the stellar halo of a galaxy of around mean size, such as the Milky Way. Deason et al found "a strikingly sharp drop in stellar density beyond *r* ~ 50 - 60 kpc" for the Milky Way [4]. The second length scale seems to refer to the radius of the Oort cloud of a Main Sequence star of around mean size, such as the sun. The outer edge of the sun's Oort cloud lies between 10,000 and 100,000 AU from the sun [5] and designates the boundary of the solar system. Both length scales (65.0 kpc and 61,000 AU) mark the boundary of the gravitational influence of the object or body (galaxy or star) on baryonic matter.

The trio of length scales: Bohr radius, reduced Compton wavelength and classical electron radius correspond through the Q/C connection to length (radius) scales pertaining to the observable universe and arguably the two primary agglomerations of matter in the 'classical' world: the galaxy and the planetary system. The planetary system is defined here as a star together with the baryonic matter that orbits it. For the sun it is the solar system.

By way of the Q/C connection, the radius of the observable universe R_{OU}, the radius R_{Gb} (the b denotes *baryonic*) of a galactic stellar halo of mean size (typified by the stellar halo of the Milky Way) and the radius R_{PS} of the planetary system of a Main Sequence star of mean size (typified by the solar system) are in ratio

$$R_{\mathrm{OU}} : R_{\mathrm{Gb}} : R_{\mathrm{PS}} \approx 1 : \alpha^{5/2} : \alpha^{5} \tag{2}$$

That is,

$$R_{\mathrm{Gb}}/R_{\mathrm{OU}} \approx \alpha^{5/2} \tag{3}$$

$$R_{\mathrm{PS}}/R_{\mathrm{OU}} \approx \alpha^{5} \tag{4}$$

Now consider a supermassive black hole of around mean size, typified by the SMBH in the centre of the Milky Way, Sgr A* of mass $M_{\mathrm{SgrA*}}$ = 4.02±0.16±0.04 x 10^6 M_{Sun} [6]. Although the spin of Sgr A* is not known, if we assume it is of low value then the radius of the supermassive black hole can be approximated by its Schwarzschild radius $R_{\mathrm{SgrA*}} = 2M_{\mathrm{SgrA*}}$. We then find that $R_{\mathrm{SgrA*}}$ (11.9 x 10^9 m) and the radius R_{H} of the Hubble sphere (14.3 x 10^9 light years in the Lambda-CDM model) are related through $R_{\mathrm{SgrA*}}/R_{\mathrm{H}} = \alpha^{7.51}$. For a SMBH of mean size, its Schwarzschild radius R_{SMBH} is given by

$$\mathrm{R_{SMBH}}/\mathrm{R_{H}} \approx \alpha^{15/2} \tag{5}$$

The quantity $\alpha^{10}.R_{\mathrm{H}}$ equals 58 km. This is the radius of the event horizon of a 20 M_{Sun} Schwarzschild black hole or a 23 M_{Sun} Kerr black hole with the typical spin parameter ($a = 0.7$) of a gravitational event remnant from the merger of two stellar black holes. Of the ten black hole merger events for which masses have been reported [7] the mean progenitor mass is 23 M_{Sun}. The mean mass of the 'secondary' progenitors, the most likely to be first generation black holes, is 20 M_{Sun}. For a stellar black hole of mean size,

$$R_{\mathrm{SBH}}/R_{\mathrm{H}} \approx \alpha^{10} \tag{6}$$

4 Mass Scales

In the Lambda-CDM model with a Hubble constant of $\mathrm{H_0}$ = 67.4 km $\mathrm{s^{-1}}$ $\mathrm{Mpc^{-1}}$ (h = 0.674) and baryonic matter density parameter $\Omega_{\mathrm{b}} =$ 0.0493 [8], the critical density of

1.878 x 10^{-26} h^2.kg.m^{-3} [9] suggests that the mass M_{Hb} of baryonic matter in the Hubble sphere is 4.36 x 10^{51} kg or $\alpha^{-9.99} M_{\mathrm{Sun}}$. The mass M_{PS} ($\approx M_{\mathrm{Sun}}$) of a planetary system of mean size is related to the mass of baryonic matter in the Hubble sphere through the equation

$$M_{\mathrm{PS}} / M_{\mathrm{Hb}} \approx \alpha^{10} \qquad (7)$$

which tells us that the baryonic mass of the Hubble sphere is unchanging The mass M_{Gb} of baryonic matter in the galaxy is approximated closely by the total stellar mass, which for the Milky Way is 4.6+2.0/-1.3 x 10^{10} M_{Sun} [10] or $\alpha^{-4.99} M_{\mathrm{Sun}}$. For a galaxy of mean size,

$$M_{\mathrm{PS}} / M_{\mathrm{Gb}} \approx \alpha^{5} \qquad (8)$$

and from (7) and (8)

$$M_{\mathrm{Gb}} / M_{\mathrm{Hb}} \approx \alpha^{5} \qquad (9)$$

For comparison with (2) we write

$$M_{\mathrm{Hb}} : M_{\mathrm{Gb}} : M_{\mathrm{PS}} \approx 1 : \alpha^{5} : \alpha^{10} \qquad (10)$$

The mass – mostly the mass of dark matter – of a galaxy is related to the total mass/energy E_{H} of the Hubble sphere. The mass of the Milky Way, a galaxy of around mean size, is $\approx 10^{12}$ M_{Sun} or $\alpha^{4.98} E_{\mathrm{H}}$. We see that the mass M_{G} of the mean-sized galaxy is given by

$$M_{\mathrm{G}} / E_{\mathrm{H}} \approx \alpha^{5} \qquad (11)$$

The mass of a Schwarzschild black hole (supermassive and stellar) of mean size is related through multiplication by an integer power of $\alpha^{5/2}$ to the total mass/energy $\mathrm{E_H} = 8.85$ x 10^{52} kg (mass equivalent) of the Hubble sphere. For the supermassive black hole Sgr A* we find that $M_{\mathrm{SgrA*}} / E_{\mathrm{H}} = \alpha^{7.51}$. For a supermassive black hole of mean size,

$$M_{\mathrm{SMBH}} / E_{\mathrm{H}} \approx \alpha^{15/2} \qquad (12)$$

From (5) and (6), the radius of a stellar black hole of mean size is related to the radius of a SMBH of mean size through the equation

$$R_{\mathrm{SBH}} / R_{\mathrm{SMBH}} \approx \alpha^{5/2} \qquad (13)$$

For black holes of low spin, the radius and mass are related through $R_{\bullet} \approx 2M_{\bullet}$ and it follows that

$$M_{\mathrm{SBH}}/M_{\mathrm{SMBH}} \approx \alpha^{5/2} \tag{14}$$

and then from (12) and (14),

$$M_{\mathrm{SBH}}/E_{\mathrm{H}} \approx \alpha^{10} \tag{15}$$

Equations (5) and (12), and also (6) and (15), require the equation $R_{\mathrm{H}} = 2E_{\mathrm{H}}$ to be valid (so that $R_{\bullet} = 2M_{\bullet}$). We find that $R_{\mathrm{H}} = 2.06 \pm 0.07\, E_{\mathrm{H}}$, the error arising mostly from uncertainty in the value of the Hubble constant. The equation

$$R_{\mathrm{H}} = 2E_{\mathrm{H}} \tag{16}$$

tells us that the radius of the Hubble sphere is also the Schwarzschild radius of the Hubble sphere. As the Hubble sphere increases in radius the total mass/energy content, E_{H}, of the sphere will increase.

5 The 'Ideal' Length and Mass Scales

The various relationships between the astrophysical and cosmological scales are summarised in Table 1.

Galactic Stellar Halo	$R_{\mathrm{Gb}}/R_{\mathrm{OU}} \approx \alpha^{5/2}$	$M_{\mathrm{Gb}}/M_{\mathrm{Hb}} \approx \alpha^{5}$
Planetary System	$R_{\mathrm{PS}}/R_{\mathrm{OU}} \approx \alpha^{5}$	$M_{\mathrm{PS}}/M_{\mathrm{Hb}} \approx \alpha^{10}$
Galaxy	-	$M_{\mathrm{G}}/E_{\mathrm{H}} \approx \alpha^{5}$
Supermassive Black Hole	$R_{\mathrm{SMBH}}/R_{\mathrm{H}} \approx \alpha^{15/2}$	$M_{\mathrm{SMBH}}/E_{\mathrm{H}} \approx \alpha^{15/2}$
Stellar Black Hole	$R_{\mathrm{SBH}}/R_{\mathrm{H}} \approx \alpha^{10}$	$M_{\mathrm{SBH}}/E_{\mathrm{H}} \approx \alpha^{10}$

Table 1: Relationships between astrophysical and cosmological scales. All astrophysical parameters refer to the geometric mean of a distribution of sizes.

R_{OU}	Radius of the observable universe
R_{H}	Radius of the Hubble sphere
M_{Hb}	Mass of baryonic matter within the Hubble sphere
E_{H}	Total mass/energy content of the Hubble sphere
R_{Gb}	Radius of a galactic stellar halo of mean size
M_{Gb}	Mass of baryonic matter bounded by a galactic stellar halo of mean size
R_{PS}	Radius of a planetary system of mean size
M_{PS}	Mass of baryonic matter in a planetary system of mean size
M_{G}	Total mass of a galaxy of mean size
R_{SMBH}	Schwarzschild radius of a supermassive black hole of mean size
M_{SMBH}	Mass of a supermassive black hole of mean size
R_{SBH}	Schwarzschild radius of a stellar black hole of mean size
M_{SBH}	Mass of a stellar black hole of mean size

The 'ideal' value – the geometric mean of a distribution of values – of each astrophysical parameter in Table 1 has been calculated with $\mathrm{R_{OU}}$ = 46.6 light years, $\mathrm{R_H}$ = 14.3 light years, $\mathrm{M_{Hb}}$ = 4.36 x 10^{51} kg and $\mathrm{E_H}$ = 8.85 x 10^{52} kg (mass equivalent); the results are presented in Table 2. 'Local' masses are shown for comparison.

	Ideal Radius	Ideal Mass	'Local' mass
Galactic Stellar Halo	65.0 kpc (212,000 lyr)	4.54 x 10^{10} M_{Sun}	4.6+2.0/-1.3 x 10^{10} M_{Sun} (Milky Way) [10]
Planetary System	61.0 x 10^3 AU (0.964 lyr)	0.939 M_{Sun}	$\approx M_{\mathrm{Sun}}$ (Solar System)
Galaxy	-	9.21 x 10^{11} M_{Sun}	$\approx 10^{12}$ M_{Sun} (Milky Way)
Supermassive Black Hole	12.7 x 10^6 km	4.19 x 10^6 M_{Sun}	4.02±0.16±0.04 x 10^6 M_{Sun} (Sgr A*) [6]
Stellar Black Hole	57.9 km	19.6 M_{Sun}	23.4 M_{Sun} (GW Events) [7]

Table 2: Values of the 'ideal' radii and masses of the galactic stellar halo, planetary system, galaxy, supermassive black hole and stellar black hole according to the equations of Table 1. The mass of the galactic stellar halo is that of baryonic matter. The local stellar black hole mass is the geometric mean of 20 gravitational wave event progenitor masses.

The Quantum/Classical connection between the Bohr radius a_0 and the radius of the observable universe ($2{a_0}^5 = {R_{\mathrm{OU}}}^2$) allows us to write down the radius of the observable universe as $R_{\mathrm{OU}} = 2^{1/2}.{a_0}^{5/2}$ and then, since $a_0 \approx (\pi/2)^{125}\, l_{\mathrm{Planck}}$ [1], as $R_{\mathrm{OU}} \approx 2^{1/2}.(\pi/2)^{625/2}\, l_{\mathrm{Planck}}$. Note that $a_0 = (\pi/2)^{125.0005}\, l_{\mathrm{Planck}}$; powers of $(\pi/2)^{25}$ feature widely in the overarching model. The radius of the observable universe and the ideal astrophysical radii are then written in terms of π, α and R_{H} in Table 3.

Observable Universe	$R_{\mathrm{OU}} \approx 2^{1/2}.(\pi/2)^{625/2}$
Galactic Stellar Halo	$R_{\mathrm{Gb}} \approx 2^{1/2}.(\pi/2)^{625/2}.\alpha^{5/2}$
Planetary System	$R_{\mathrm{PS}} \approx 2^{1/2}.(\pi/2)^{625/2}.\alpha^{5}$
Supermassive Black Hole	$R_{\mathrm{SMBH}} \approx \alpha^{15/2}.R_{\mathrm{H}}$
Stellar Black Hole	$R_{\mathrm{SBH}} \approx \alpha^{10}.R_{\mathrm{H}}$

Table 3: The radius of the observable universe and the ideal radii of the galaxy, planetary system, supermassive black hole and stellar black hole in Planck units.

R_{OU} Radius of the observable universe
R_{H} Radius of the Hubble sphere
R_{Gb} Radius of a galactic stellar halo of mean size

In the limit as $R_{\mathrm{H}} \longrightarrow R_{\mathrm{OU}}$ the five length scales of Table 3 will lie in geometric progression.

6 Black Holes in an expanding universe

Equation (5): $R_{\mathrm{SMBH}}/R_{\mathrm{H}} \approx \alpha^{15/2}$ tells us that as space expands the supermassive black hole will grow in tandem with the Hubble sphere, i.e. at a rate $\dot{R}_{\bullet}/R_{\bullet} \approx H$, presumably by accretion and mergers. Debattista et al have shown that SMBH growth must occur in spiral galaxies as discs assemble [11]. Their results suggest that Sgr A* is gaining mass at a rate approximately equal to one solar mass per 3,000 years and that the supermassive black hole of mass 6.6±0.4 x 10^{8} M_{Sun} in NGC 4594 [12] is gaining mass at a rate approximately equal to one solar mass per 20 years. These two values of mass growth rate are equivalent to radius growth rates $\dot{R}_{\bullet}/R_{\bullet}$ of 2.6 x 10^{-18} s^{-1} and 2.4 x 10^{-18} s^{-1}, respectively, that are consistent with the expansion rate of space, $H_0 = 67.4$ km s^{-1} Mpc^{-1} or 2.2 x 10^{-18} s^{-1}.

7 Stars

The quantity $R_*/2M_*$ compares the radius R_* of the star to its Schwarzschild radius $R_{\mathrm{S},*} = 2M_*$. Using Planck units, we find that $R_{\mathrm{Sun}}/2M_{\mathrm{Sun}} = \alpha^{-2.51}$. For the next six nearest (Main Sequence) stars with radii in the range 0.7 – 2.0 R_{Sun} the values of $R_*/2M_*$ are $\alpha^{-2.54}$ (Alpha Centauri A), $\alpha^{-2.50}$ (Alpha Centauri B), $\alpha^{-2.48}$ (Sirius A), $\alpha^{-2.49}$ (Epsilon Eridani), $\alpha^{-2.52}$ (Tau Ceti) and $\alpha^{-2.51}$ (Epsilon Indi). The stellar radius and mass data are provided in [13]. For the ideal star of mass $M_{*\mathrm{i}} \approx M_{\mathrm{PS}}$ (= 0.939 M_{Sun}) and radius $R_{*\mathrm{i}}$,

$$R_{*\mathrm{i}}/2M_{*\mathrm{i}} \approx \alpha^{-5/2} \tag{17}$$

and therefore

$$R_{\mathrm{S},*\mathrm{i}}/R_{*\mathrm{i}} \approx \alpha^{5/2} \tag{18}$$

The Schwarzschild radius of the ideal star divided by the radius of the star equals $\alpha^{5/2}$. Since $M_{*\mathrm{i}} \approx M_{\mathrm{PS}}$ and $M_{\mathrm{PS}}/M_{\mathrm{Hb}} \approx \alpha^{10}$ it follows from (17) that

$$R_{*\mathrm{i}} \approx \alpha^{15/2}.2M_{\mathrm{Hb}} \tag{19}$$

which is of value 0.876 R_{Sun}. It is interesting to compare (19) with (5): $\mathrm{R_{SMBH}}/\mathrm{R_H} \approx \alpha^{15/2}$, which, since $R_{\mathrm{H}} \approx 2E_{\mathrm{H}}$, we can rewrite as

$$\mathrm{R_{SMBH}} \approx \alpha^{15/2}.2E_{\mathrm{H}} \tag{20}$$

Equations (19) and (20) show an arresting relationship between the radius of the ideal star, whose mass is baryonic, and the Schwarzschild radius of the ideal supermassive black hole, whose mass is generic:

$$R_{*\mathrm{i}}/\mathrm{R_{SMBH}} \approx M_{\mathrm{Hb}}/E_{\mathrm{H}} \tag{21}$$

The ideal star is of mass ≈0.939 M_{Sun} and radius ≈0.876 R_{Sun}. Our neighbour Alpha Centauri B, of mass 0.9373±0.0033 M_{Sun} [14] and radius 0.8632±0.0037 R_{Sun} [15], may be ideal, at least with regard to its mass and radius. We have already seen that for this star $R_*/2M_* = \alpha^{-2.50}$, which seems to be the ideal value for a geometric-mean-sized Main Sequence star. Writing the Quantum/Classical connection of (1) in the form

$$2{m_{\mathrm{Q}}}^{-5} = {R_*}^2 \tag{22}$$

and with $R_* = 0.8632\ R_{\mathrm{Sun}}$ one finds that the corresponding 'quantum' mass m_{Q} is of value 52.3±0.1 GeV or 56.1±0.1 atomic mass units. The radius of Alpha Centauri B corresponds to the mass of a nuclide of mass number A = 56. Similarly, the radius of the Sun corresponds to a 'quantum' mass of 53.0 atomic mass units and the radius of Alpha Centauri A corresponds to a 'quantum' mass of 48.9±0.1 atomic mass units [16]. The only stable nuclides with A = 56, 53 or 49 are $^{56}Fe_{26}$, $^{53}Cr_{24}$ and $^{49}Ti_{22}$. The corresponding quantum length scales r_{Q} are calculated from

$$2{r_{\mathrm{Q}}}^{5} = (M_*/\alpha m_{\mathrm{e}})^2 \tag{23}$$

where $\alpha m_e = {a_0}^{-1}$. The length scales r_Q corresponding to the masses M_* of Alpha Centauri B, the Sun and Alpha Centauri A are 134 pm, 138 pm and 143 pm, respectively. The empirical atomic radii of iron, chromium and titanium are all quoted as 140 pm [17] with an uncertainty of around 5%. There is an increasing trend in atomic radius with decreasing Z across Period 4, which is consistent with our results. We see that Alpha Centauri B, the Sun and Alpha Centauri A are the counterparts of the nuclides $^{56}Fe_{26}$, $^{53}Cr_{24}$ and $^{49}Ti_{22}$, respectively. The 'ideal star' Alpha Centauri B is the counterpart of the nuclide, $^{56}Fe_{26}$, that has the lowest mass per nucleon of any nuclide.

9 Conclusions

- The radius of the observable universe is unchanging.
- The baryonic mass of the Hubble sphere is unchanging.
- The radius of the Hubble sphere is also the Schwarzschild radius of the Hubble sphere. The total mass/energy of the Hubble sphere increases with time.
- The stellar halo radius, baryonic mass and total mass of the Milky Way, and the radii and masses of the solar system, sun and supermassive black hole Sgr A* all take near-ideal values, i.e. geometric mean values of the distributions of values at z ~ 0.
- The radius of the ideal galactic stellar halo equals the radius of the observable universe multiplied by $\alpha^{5/2}$. Value: 65 kpc.
- The baryonic mass of the ideal galaxy equals the baryonic mass of the Hubble sphere multiplied by α^5. Value: 4.5 x 10^{10} M_{Sun}.
- The total mass, including dark matter, of the ideal galaxy equals the total mass/energy of the Hubble sphere multiplied by α^5 and increases at Hubble rate. Value: 9.2 x 10^{11} M_{Sun}.
- The radius of the ideal planetary system equals the radius of the observable universe multiplied by α^5. Value: 0.96 lyr.
- The mass of the ideal planetary system (≈ mass of the ideal star) equals the baryonic mass of the Hubble sphere multiplied by α^{10}. Value: 0.94 M_{Sun}.
- The Schwarzschild radius of the ideal supermassive black hole equals the radius of the Hubble sphere multiplied by $\alpha^{15/2}$ and increases at Hubble rate. The mass of the ideal supermassive black hole equals the total mass/energy of the Hubble sphere multiplied by $\alpha^{15/2}$; value: 4.2 x 10^6 M_{Sun}.
- The Schwarzschild radius of the ideal stellar black hole equals the radius of the Hubble sphere multiplied by α^{10} and increases at Hubble rate. The mass of the ideal stellar black hole equals the total mass/energy of the Hubble sphere multiplied by α^{10}; value: 20 M_{Sun}.
- The Schwarzschild radius of the ideal star equals the radius of the star multiplied by $\alpha^{5/2}$.

- In Planck units, the radius of the ideal star equals the baryonic mass of the Hubble sphere multiplied by $2\alpha^{15/2}$. Value: 0.88 R_{Sun}.
- The radius of the ideal star divided by the Schwarzschild radius of the ideal supermassive black hole equals the baryonic mass of the Hubble sphere divided by the total mass/energy of the Hubble sphere.
- By way of the Quantum/Classical connection, the radius of a mid-Main Sequence star maps onto the atomic mass of a specific period 4 nuclide while the mass of the star maps onto the atomic radius of the same nuclide. The ideal star is the counterpart of the nuclide ^{56}Fe. The sun is the counterpart of the nuclide ^{53}Cr.

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