Refutation of the Russell-Prawitz embedding

Abstract: We evaluate the Russell-Prawitz embedding as *not* tautologous. Hence atomization of universal instantiation does not follow (nor does proof reduction, weakening of dinaturality conversion, or strict simulation). These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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1 Introduction The Russell-Prawitz translation of the intuitionistic propositional calculus **IPC** into second-order intuitionistic propositional calculus **NI**^{2} , the latter based on the language only containing implication, conjunction and the second-order universal quantifier, rests on the following enco[d]ing of disjunction and absurdity

$$A \vee B := \forall X.((A \supset X) \wedge (B \supset X)) \supset X \text{ and } \bot := \forall X.X. \qquad (1.1.1)$$

LET p, q, r: A, B, X.

((p+q)=(((p>#r)&(q>#r))>r))&((r=r)=#r) ;
FFFF **F**`NNN` **FFFF** **F**`NNN` (1.1.2)

Remark 1.1.2: Eq. 1.1.2 is *not* tautologous, hence refuting the Russell-Prawitz embedding and therefore atomization of universal instantiation.

However in an effort to resuscitate the conjecture we present the truth table value results for the antecedent and consequent in Eq. 1.1.2:

(p+q)=(((p>#r)&(q>#r))>r) ; `TTTT` **F**`TTT` `TTTT` **F**`TTT` (1.1.2.1.2)

(r=r)=#r ; **FFFF** `NNNN` **FFFF** `NNNN` (1.1.2.2.2)

The antecedent and consequent of Eq. 1.1.2 are also *not* tautologous.