Refutation of rewriting logic for compositional specification

Abstract: We evaluate the first motivational example, before mutual exclusion of multiple trains, for states and transitions as defined. The conjectured model is *not* tautologous, refutes rewriting logic for compositional specification, and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

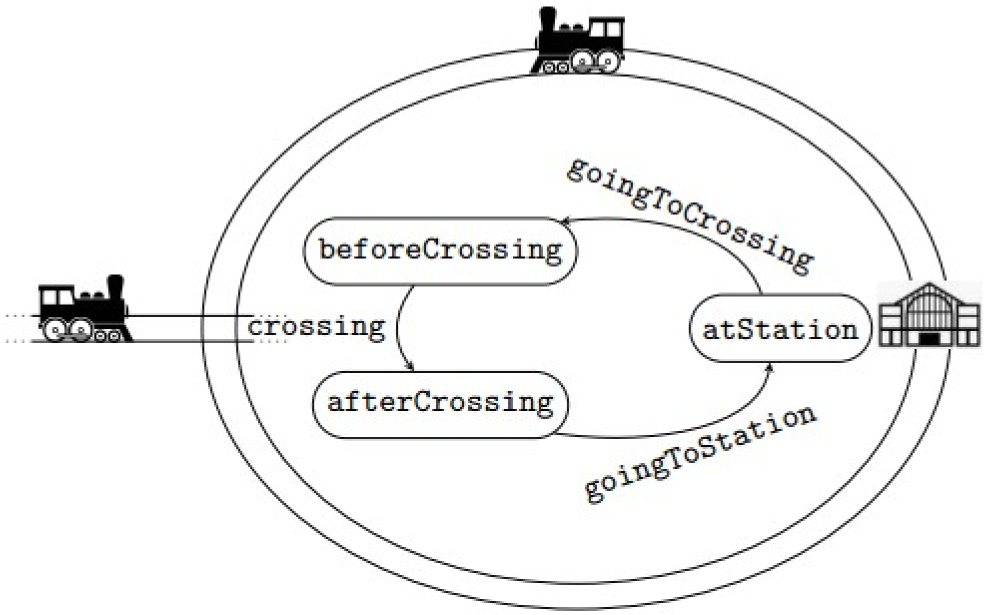
From: Martín, Ó.; Verdejo, A.; Martí-Oliet, N. (2003). Compositional specification in rewriting logic. arxiv.org/pdf/1908.11769.pdf omartins@ucm.es

Abstract Rewriting logic is naturally concurrent: several subterms of the state term can be rewritten simultaneously. But state terms are global, which makes compositionality difficult to achieve. Compositionality here means being able to decompose a complex system into its functional components and code each as an isolated and encapsulated system. Our goal is to help bringing compositionality to system specification in rewriting logic. The base of our proposal is the operation that we call synchronous composition. We discuss the motivations and implications of our proposal, formalize it for rewriting logic and also for transition structures, to be used as semantics, and show the power of our approach with some examples.

2 Motivation, goals, and choices

2.1 First motivational example: mutual exclusion

Think of a train, a very simple model of a train, that goes round a closed railway in which there is a station and a crossing with another railway. There are three points of interest in the railway, that we use as the states of our model. There are three transitions for moving between the three states.



In Maude-like notation:

```
rl [goingToCrossing]    : atStation       => beforeCrossing .
rl [crossing]           : beforeCrossing  => afterCrossing .
rl [goingToStation]     : afterCrossing   => atStation .
```

The keyword `rl` introduces a rewrite rule. The identifier in square brackets is the label of the rule. Rules describe transitions between states. To the left of the arrow (=>) is the origin state; to the right is the destination state.

LET	u, v, w:	[states]	station, before_crossing, after_crossing;
	x, y, z:	[transitions]	to_crossing, crossing, to_station.

Remark 12.1: We write the modeled conjecture to mean the circular states of (u>v>w>u) imply the transition definitions of (x&y&z). (12.1.0)

(((x>y)>(z>x))&((u>v)>(w>u)))>(((x=(u>v))&(y=(v>w)))&(z=(w>u))) ;

```
FFFF FFFF FFFF FFFF( 8)      }
TTTT TTTT TTTT TTTT( 2) }x2  }x4
FFFF FFFF FFFF FFFF( 2) }    }
TTTT TTTT TTTT TTTT(16)
FFFF FFFF FFFF FFFF( 4)
TTTT TTTT TTTT TTTT( 6)
FFFF FFFF FFFF FFFF( 2)
TTTT TTTT TTTT TTTT( 2)
FFFF FFFF FFFF FFFF( 2)
TTTT TTTT TTTT TTTT(18)
FFFF FFFF FFFF FFFF( 6)
TTTT TTTT TTTT TTTT( 2)
FFFF FFFF FFFF FFFF( 2)
TTTT TTTT TTTT TTTT( 4)
```
(12.1.2)

Eq. 12.1.2 is *not* tautologous. This denies the first motivational example, before mutual exclusion is invoked for multiple trains, and hence refutes the conjecture of rewriting logic for compositional specification.