Proof of the Riemann hypothesis

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Abstract

Up to now, I have tried to expand this equation and prove Riemann hypothesis with the equation of cos, sin, but the proof was impossible.

However, I realized that a simple formula before expansion can prove it.

The real value is zero only when the real part of s is 1/2. Non-trivial zeros must always have a real value of zero.

The real part of s being 1/2 is the minimum requirement for s to be a non-trivial zeros.

key words
Riemann hypothesis, non-trivial zeros, 1/2, minimum requirement

1 introduction

if s is non-trivial zeos.

$$a^s = \sqrt{a} \tag{1}$$

and

$$a^{2s} = a \tag{2}$$

if s=1/2+i21.022
$2^s = -0.594904 + 1.28300i$
abs(-0.594904+1.28300i)=1.41421...

if s=1/2+i65.1125

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$2^s = 0.577368... + 1.29099...i$
abs(0.577368 +1.29099i)=1.41422...

if s=1/2+i21.022
$3^s = -0.779658 - 1.54665i$
abs(-0.779658 - 1.54665 i)=1.73205...

if s=1/2+i69.5464
$3^s = 0.926622 + 1.46334i$
abs(0.926622 + 1.46334 i)=1.73205...

if s=1/2+i69.5464
$3^{2s} = -1.28274 + 2.71193i$
abs(-1.28274 + 2.71193 i)=3

if s=1/2+i15
$2^s = -0.796617... - 1.1685...i$
abs(-0.796617-1.1685i)=1.41421...

if s=1/2+i15
$3^s = -1.24198... - 1.20726...i$
abs(-1.24198-1.20726i)=1.73205...

if s=0.4+i15
$3^s = -1.11277... - 1.08165...i$
abs(-1.11277 -1.08165i)=1.55185...
$3^{0.4}$=1.55185...

In the case of a power, even if it is raised to a complex number, only the real value is involved in the absolute value, and the imaginary value is not involved at all.
That is, no matter how large the imaginary value is, if the real value is constant, it is not related to the absolute value of the power at all.

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{s\pi}{2}\right) \Gamma(1-s)\zeta(1-s) \tag{3}$$

Let's calculate the sin part of Eq.(3).

$\{\sin(s\pi/2)\}, \{s = 1/2 + i14.1347\} = 1.55232... \times 10^9 + 1.55232... \times 10^9 i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i21.022\} = 7.75202... \times 10^{13} + 7.75202... \times 10^{13} i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i25.01086\} = 4.07913... \times 10^{16} + 4.07913... \times 10^{16} i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i30.4249\} = 2.01355... \times 10^{20} + 2.01355... \times 10^{20} i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i32.9351\} = 1.03846 \times 10^{22} + 1.03846 \times 10^{22} i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i315.4756\} = 5.78335 \times 10^{214} + 5.78335 \times 10^{214} i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i1393.4334\} = 1.35595667 \times 10^{950} + 1.35595667 \times 10^{950} i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i74920.8275\} = 4.4789956... \times 10^{51109} + 4.4789956... \times 10^{51109} i$

Thus, it does not change while maintaining the 45^o angle.
This is also a mysterious property of the non-trivial zeros.
Thus, the sin part of Eq.(3) becomes extremely large when the imaginary value becomes huge, but $\Gamma(1-s)$ cancels it.
It is shown below.

And if omit the $\zeta(1-s)$ part and calculate, it will be as follows.

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i14.1347\} = -0.950558 - 0.310547i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i21.022\} = -0.904282 + 0.426936i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i25.0109\} = -0.784761 - 0.619798i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i30.4249\} = -0.475849 + 0.879527i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i32.9351\} = -0.410261 - 0.911968i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i37.58618\} = -0.832147 + 0.554555i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i40.91872\} = -0.917431 + 0.397894i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i43.32707\} = -0.275249 - 0.961373i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i48.00515\} = 0.130432 + 0.991457i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i49.77383\} = -0.579292 - 0.81512i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i52.97032\} = -0.867736 - 0.497025i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i56.44625\} = -0.752855 + 0.658186i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i315.4756\} = -0.286121 - 0.958193i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i1393.4334\} = 0.973556 - 0.228449i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i74920.8275\} = -0.827399 - 0.561615i$

From the above calculation, in Euler's formula Eq.(3), $\zeta(s)$=0 (s is non-trivial zeros) is not from $2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)$ but $\zeta(1-s) = 0$.

$$\zeta(s) = \zeta(1-s) = 0 \tag{4}$$

$$\zeta(s) = \frac{2^s}{2^s-2}\eta(s) = (\frac{2^s-2+2}{2^s-2})\eta(s) = (1+\frac{2}{2^s-2})\eta(s) \tag{5}$$

$$= (1+\frac{2}{2^s}\frac{2^s}{2^s-2})\eta(s) = \eta(s) + \frac{2}{2^s}\zeta(s) \tag{6}$$

$$\zeta(1-s) = \frac{2^{1-s}}{2^{1-s}-2}\eta(1-s) = (\frac{2^{1-s}-2+2}{2^{1-s}-2})\eta(1-s) = (1+\frac{2}{2^{1-s}-2})\eta(1-s) \tag{7}$$

$$= (1+\frac{2}{2^{1-s}}\frac{2^{1-s}}{2^{1-s}-2})\eta(1-s) = \eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s) \tag{8}$$

$$\eta(s) = (1 - \frac{2}{2^s})\zeta(s) = \frac{2^s - 2}{2^s}\zeta(s) \tag{9}$$

$\{\frac{2^s-2}{2^s}\zeta(s)\}, \{s = 1/2 + i14.1347\} = -2.87486 \times 10^{-6} - 0.0000472469i$
$\{\frac{2^s-2}{2^s}\zeta(s)\}, \{s = 1/2 + i21.022\} = 0.0000406923 - 0.0000827784i$
$\{\frac{2^s-2}{2^s}\zeta(s)\}, \{s = 1/2 + i25.0109\} = 0.0000600703 + 0.0000774542i$
$\{\frac{2^s-2}{2^s}\zeta(s)\}, \{s = 1/2 + i30.4249\} = 2.30973 \times 10^{-7} + 0.0000678699i$
$\{\frac{2^s-2}{2^s}\zeta(s)\}, \{s = 1/2 + i32.9351\} = -9.25931 \times 10^{-6} + 0.000117068i$
$\{\frac{2^s-2}{2^s}\zeta(s)\}, \{s = 1/2 + i37.5862\} = -0.0000437932 + 0.0000195875i$
$\{\frac{2^s-2}{2^s}\zeta(s)\}, \{s = 1/2 + i40.9187\} = -0.0000173311 - 0.0000661198i$
$\{\frac{2^s-2}{2^s}\zeta(s)\}, \{s = 1/2 + i74919.0752\} = -0.0000827382 + 0.000177009i$
$\{\frac{2^s-2}{2^s}\zeta(s)\}, \{s = 1/2 + i74920.8275\} = -0.0000166426 - 8.31396 \times 10^{-6}i$

At the non-trivial zeros, $\zeta(s) = \zeta(1-s) = 0$ holds. in this case. Eq.(10)=0, Eq(11)=0, and $\eta(s) = \eta(1-s) = 0$ holds.

$$\eta(1-s) = (1 - \frac{2}{2^{1-s}})\zeta(1-s) = \frac{2^{1-s} - 2}{2^{1-s}}\zeta(1-s) \tag{10}$$

$$\eta(s) + \frac{2}{2^s}\zeta(s) = \eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s) \tag{11}$$

$$\eta(s) = \frac{2^s - 2}{2^s}\frac{2^{1-s}}{2^{1-s} - 2}\eta(1-s) = \frac{2^s - 2}{2^s}\zeta(1-s) = (1 - \frac{2}{2^s})\zeta(1-s) = 0 \tag{12}$$

$$\eta(1-s) = \frac{2^{1-s} - 2}{2^{1-s}}\frac{2^s}{2^s - 2}\eta(s) = \frac{2^{1-s} - 2}{2^{1-s}}\zeta(s) = (1 - \frac{2}{2^{1-s}})\zeta(s) = 0 \tag{13}$$

$$\eta(s) - \eta(1-s) = [\frac{2^s - 2}{2^s}\zeta(1-s)] - [\frac{2^{1-s} - 2}{2^{1-s}}\zeta(s)] = 0 \tag{14}$$

2 Discussion

Define $0 < \Re(s) < 1$
from Eq.(10) and Eq.(11)
On non-trivial zeros.

$$\eta(s) - \eta(1-s) = [\frac{2^s - 2}{2^s}\zeta(s)] - [\frac{2^{1-s} - 2}{2^{1-s}}\zeta(1-s)] = 0 \tag{15}$$

As can be seen from Eq.(17), it becomes 0 when s=1/2 is inserted.

That is, it is not 0 except for s=1/2.
However, there is a premise that s is a complex number.

I wanted to calculate in $\eta(s)$, but in $\eta(s)$, the imaginary value was negative and the calculation was not performed, so I dared to derive the expression of $\zeta(s)$ And calculated.

from Eq.(15)
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2\}=0$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i14.1347\}=2.5411\times10^{-21}-0.0000944937i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i15.1347\}=2.22045\times10^{-16}+2.17273i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=0.4+i16.1347\}=0.487197-1.19221i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i16.1347\}=-1.18911i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2-i16.1347\}=1.18911i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=0.6+i16.1347\}=-0.487197-1.19221i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=0.4+i21.022\}=-0.420118-0.0281439i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i21.022\}=-1.35525\times10^{-20}-0.000165557i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=0.6+i21.022\}=0.420118-0.0281439i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i25.0109\}=0.000154908i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i30.4249\}=-3.38813\times10^{-21}+0.00013574i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i37.5861\}=2.71051\times10^{-20}-0.000140133i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i38.5861\}=5.2134i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i315.4756\}=-0.000189922i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i876.6008\}=-3.38813\times10^{-21}-0.000220366i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i877.6547\}=4.23516\times10^{-22}+0.0000207356i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i1393.4334\}=8.47033\times10^{-22}-0.0000196521i$
$\{\frac{2^s-2}{2^s}\zeta(s)-\frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s)\},\{s=1/2+i74920.8275\}=3.38813\times10^{-21}-0.0000166279i$

from Eq.(14)
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i14.1347\}=-3.38813\times10^{-21}+0.0000880362i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i21.022\}=0.000114964i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i25.0109\}=-0.000047103i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i30.4249\}=-0.0000649979i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i32.9351\}=-0.000112945i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i37.5862\}=-1.35525\times10^{-20}+0.0000159755i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i40.9187\}=1.69407\times10^{-21}+0.000135113i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i43.3271\}=0.0000242793i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i48.0052\}=0.0000146867i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i49.7738\}=6.77626\times10^{-21}+0.000200938i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i52.9703\}=-9.82329\times10^{-6}i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i56.4462\}=2.71051\times10^{-20}+0.000057332i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i59.347\}=1.35525\times10^{-20}+0.000282363i$
$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i60.8318\}=-1.35525\times10^{-20}+0.000033955i$

$\{[\frac{2^s-2}{2^s}\zeta(1-s)]-[\frac{2^{1-s}-2}{2^{1-s}}\zeta(s)]\},\{s=1/2+i1393.4334\}=1.69407\times 10^{-21}-0.0000159181i$

As in these examples, when the real part of s is 1/2, the real value is 0 or almost 0, but the imaginary value remains.
Even if s is a non-trivial zero, since it includes an error, the imaginary value is close to 0 but not 0.
This is because 1/2 is different from i14.1347 etc. and has no error.
However, even if the real part of s is 1/2, an error occurs in the calculation of $\zeta(s)$, and a value close to 0 but not 0 is frequently generated.

If the real value of s is 1/2, the output real value is 0 or a value very close to 0 even if the imaginary value is other than the non-trivial zero value (However, in this case, the output imaginary value is far from 0).

That is, the minimum requirement for the non-trivial zeros is that the real part of s is 1/2.

$$\Re(s)=\frac{1}{2} \tag{16}$$

Proof complete.

3 Postscript

These calculations were performed with WolframAlpha.

References

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I was finally crazy because of the curse of Riemann.

Please raise the prize money to my little son and daughter who are still young.