Proof of the Riemann hypothesis

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Abstract

Up to now, I have tried to expand this equation and prove Riemann hypothesis with the equation of cos, sin, but the proof was impossible.

However, I realized that a simple formula before expansion can prove it.

The real value is zero only when the real part of s is 1/2. Non-trivial zeros must always have a real value of zero.

The real part of s being 1/2 is the minimum requirement for s to be a non-trivial zeros.

key words
Riemann hypothesis, non-trivial zeros, 1/2, minimum requirement

1 introduction

if s is non-trivial zeos.

$$a^s = \sqrt{a} \tag{1}$$

and

$$a^{2s} = a \tag{2}$$

if s=1/2+i21.022
$2^s = -0.594904 + 1.28300i$
abs(-0.594904+1.28300i)=1.41421...

if s=1/2+i65.1125

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$2^s = 0.577368... + 1.29099...i$
abs(0.577368 +1.29099i)=1.41422...

if s=1/2+i21.022
$3^s = -0.779658 - 1.54665i$
abs(-0.779658 - 1.54665 i)=1.73205...

if s=1/2+i69.5464
$3^s = 0.926622 + 1.46334i$
abs(0.926622 + 1.46334 i)=1.73205...

if s=1/2+i69.5464
$3^{2s} = -1.28274 + 2.71193i$
abs(-1.28274 + 2.71193 i)=3

if s=1/2+i15
$2^s = -0.796617... - 1.1685...i$
abs(-0.796617-1.1685i)=1.41421...

if s=1/2+i15
$3^s = -1.24198... - 1.20726...i$
abs(-1.24198-1.20726i)=1.73205...

if s=0.4+i15
$3^s = -1.11277... - 1.08165...i$
abs(-1.11277 -1.08165i)=1.55185...
$3^{0.4}$=1.55185...

In the case of a power, even if it is raised to a complex number, only the real value is involved in the absolute value, and the imaginary value is not involved at all.
That is, no matter how large the imaginary value is, if the real value is constant, it is not related to the absolute value of the power at all.

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{s\pi}{2}\right)\Gamma(1-s)\zeta(1-s) \tag{3}$$

Let's calculate the sin part of Eq.(3).

$\{\sin(s\pi/2)\}, \{s = 1/2 + i14.1347\} = 1.55232... \times 10^9 + 1.55232... \times 10^9 i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i21.022\} = 7.75202... \times 10^{13} + 7.75202... \times 10^{13} i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i25.01086\} = 4.07913... \times 10^{16} + 4.07913... \times 10^{16} i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i30.4249\} = 2.01355... \times 10^{20} + 2.01355... \times 10^{20} i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i32.9351\} = 1.03846 \times 10^{22} + 1.03846 \times 10^{22} i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i315.4756\} = 5.78335 \times 10^{214} + 5.78335 \times 10^{214} i$
$\{\sin(s\pi/2)\}, \{s = 1/2 + i1393.4334\} = 1.35595667 \times 10^{950} + 1.35595667 \times 10^{950} i$

Thus, it does not change while maintaining the 45^o angle.
This is also a mysterious property of the non-trivial zeros.
Thus, the sin part of Eq.(3) becomes extremely large when the imaginary value becomes huge, but $\Gamma(1-s)$ cancels it.
It is shown below.

And if omit the $\zeta(1-s)$ part and calculate, it will be as follows.

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i14.1347\}=-0.950558-0.310547i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i21.022\}=-0.904282+0.426936i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i25.0109\}=-0.784761-0.619798i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i30.4249\}=-0.475849+0.879527i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i32.9351\}=-0.410261-0.911968i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i37.58618\}=-0.832147+0.554555i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i40.91872\}=-0.917431+0.397894i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i43.32707\}=-0.275249-0.961373i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i48.00515\}=0.130432+0.991457i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i49.77383\}=-0.579292-0.81512i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i52.97032\}=-0.867736-0.497025i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i56.44625\}=-0.752855+0.658186i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i315.4756\}=-0.286121-0.958193i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\},\{s=1/2+i1393.4334\}=0.973556-0.228449i$

$In the calculation above, Euler's formula Eq.(3) has a non-trivial zero value of \zeta(s)=0.$
The other element, $2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)$, indicates that $\zeta(s)=0$ is not useful.
It is $\zeta(1-s)=0$ that is useful for $\zeta(s)=0$.
That is, from Eq.(3).

$$\zeta(s)=\zeta(1-s)=0 \tag{4}$$
$$\eta(s)=\eta(1-s)=0 \tag{5}$$

This is clear from $\zeta(s)=\frac{2^s}{2^s-2}\eta(s)$, that Eq.(4) and Eq.(5) have the same significance.
Both equations are valid only for non-trivial zeros.

In the case of $\eta(s)$, the proof of Riemann hypothesis is completed if it is proved that the value of the non-trivial zeros is taken only when the real part is 1/2.

Define $\quad 0<\Re(s)<1$

$$\eta(s)=\frac{2^s-2}{2^s}\zeta(s)=\zeta(s)-\frac{2}{2^s}\zeta(s) \tag{6}$$

2 Discussion

from Eq.(6)

$$\zeta(s) = \eta(s) + \frac{2}{2^s}\zeta(s) \tag{7}$$

and

$$\zeta(1-s) = \eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s) \tag{8}$$

from $\zeta(s) = \zeta(1-s)$

$$[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)] = 0 \tag{9}$$

$$[\eta(s) - \eta(1-s)] + [\frac{2}{2^s}\zeta(s) - \frac{2}{2^{1-s}}\zeta(1-s)] = 0 \tag{10}$$

$$[\eta(s) - \eta(1-s)] + [2^{1-s}\zeta(s) - 2^s\zeta(1-s)] = 0 \tag{11}$$

As can be seen from Eq.(11), it becomes 0 when s=1/2 is inserted.
That is, it is not 0 except for s=1/2.
However, there is a premise that s is a complex number.
However, $s = 1/2$ can also be expressed as $s = 1/2 + i0$.

from Eq.(9)
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 1/2 + i0\} = 0$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 0.4 + i16.1347\} = -0.493359... + 3.65957...i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 0.4 - i16.1347\} = -0.493359... - 3.65957...i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 1/2 + i14.1347\} = 0.000055107...i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 1/2 + i16.1347\} = 3.64713...i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 0.6 + i16.1347\} = 0.493359 + 3.65957...i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 0.6 - i16.1347\} = 0.493359... - 3.65957...i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 1/2 + i17.1347\} = 5.39992...i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 1/2 + i21.022\} = 0.000077614...i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 1/2 + i22.022\} = -2.61712...i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 1/2 - i22.022\} = 2.61712...i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 0.4 - i999999.1347\} = -1.85955 - 5.38649i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 1/2 + i999999.1347\} = 3.54778i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 1/2 - i999999.1347\} = -3.54778i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 1/2 + i999922.022\} = -5.59975i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 1/2 - i999922.022\} = 5.59975i$
$\{[\eta(s) + \frac{2}{2^s}\zeta(s)] - [\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)]\}, \{s = 0.6 - i999922.022\} = -10.9232 + 4.41554i$

As in these examples, when the real part is 1/2, the real value is 0, but the imaginary value remains.
When the real part of s is 1/2, the real value is completely 0,
however, even if the imaginary value of s is a non-trivial zero, an ordinary computer always includes

an error in the non-trivial zero value.
Therefore, even if s is a non-trivial zero, since it includes an error, the imaginary value is close to 0 but not 0.
This is because 1/2 is different from i14.1347 and has no error.

The real part of s is 1/2 is the minimum requirement for real value of $\zeta(s)$ and real value of $\zeta(1-s)$ are equal.(However, if the imaginary part is not non-trivial zeros value, it will not be zero.)
And,
the real part of s is 1/2 is the minimum requirement for real value of $\eta(s)$ and real value of $\eta(1-s)$ are equal.(However, if the imaginary part is not non-trivial zeros value, it will not be zero.)
That is, the minimum requirement for the non-trivial zeros is that the real part of s is 1/2.

That is, the real value is 0 only when the real part of s is 1/2.
Non-trivial zeros must always have a real value of 0.

$$\Re(s) = \frac{1}{2} \tag{12}$$

Proof complete.

3 Postscript

These calculations were performed with WolframAlpha.

References

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I was finally crazy because of the curse of Riemann.

Please raise the prize money to my little son and daughter who are still young.