Reissner-Nodstrom Solution and Energy-Momentum Density's Conservation Law

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ABSTRACT

We find Reissner-Nodstrom solution hold by the energy-momentum density's conservation law (Noether's theorem) of electromagnetic field in general relativity theory.

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Key words:General relativity theory;

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1. Introduction

Our article's aim is that we find Reissner-Nodstrom solution hold by the energy-momentum density's conservation law (Noether's theorem) of electromagnetic field.

In general relativity theory, the energy-momentum tensor $T^{\mu\upsilon}$ of the electromagnetic field is

$$T^{\mu\upsilon} = \frac{1}{4\pi c}(F^{\mu}{}_{\rho}F^{\upsilon\rho} - \frac{1}{4}g^{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma}) \tag{1}$$

In this time, spherical coordinates is in general relativity theory

$$ds^2 = -A(r,t)dt^2 + B(r,t)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{2}$$

Hence, Faraday tensors $F^{\mu\upsilon}, F_{\mu\upsilon}$ are in general relativity theory.

$$F^{\mu\upsilon} = \begin{pmatrix} 0 & -E(r,t) & 0 & 0 \\ E(r,t) & 0 & 0 & 0 \\ 0 & 0 & 0 & -B(t,r) \\ 0 & 0 & B(t,r) & 0 \end{pmatrix} \tag{3-i}$$

$$F_{\mu\upsilon} = \begin{pmatrix} 0 & E(t,r) & 0 & 0 \\ -E(t,r) & 0 & 0 & 0 \\ 0 & 0 & 0 & -B(t,r) \\ 0 & 0 & B(t,r) & 0 \end{pmatrix} \tag{3-ii}$$

Hence, Einstein-Maxwell equations is

$$R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}R = -\frac{8\pi G}{c^4}T_{\mu\upsilon} = -\frac{2G}{c^5}(F_{\mu\rho}F_{\upsilon}{}^{\rho} - \frac{1}{4}g_{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma}) \tag{4}$$

$$F^{\mu\upsilon}{}_{;\upsilon} = 0 \tag{5}$$

$$\frac{\partial F_{\mu\upsilon}}{\partial x^{\rho}} + \frac{\partial F_{\upsilon\rho}}{\partial x^{\mu}} + \frac{\partial F_{\rho\mu}}{\partial x^{\upsilon}} = 0 \tag{6}$$

2. Reissner-Nodstrom solution and Noether's theorem

In this time, energy-momentum tensor $T^{\mu\upsilon}$ is

$$\begin{aligned} T^{\mu\upsilon} &= \frac{1}{4\pi c}(F^{\mu}{}_{\rho}F^{\upsilon\rho} - \frac{1}{4}g^{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma}) \\ &= \frac{1}{4\pi c}(g^{\mu\upsilon}F_{\upsilon\rho}F^{\upsilon\rho} - \frac{1}{4}g^{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma}) \end{aligned} \tag{7}$$

Hence, $T^{\mu\upsilon}$ is

$$T^{\mu\upsilon} = \frac{E^2+B^2}{8\pi c}\begin{pmatrix} -g^{00} & 0 & 0 & 0 \\ 0 & -g^{11} & 0 & 0 \\ 0 & 0 & g^{22} & 0 \\ 0 & 0 & 0 & g^{33} \end{pmatrix} = \frac{E^2+B^2}{8\pi c}\begin{pmatrix} 1/A & 0 & 0 & 0 \\ 0 & -1/B & 0 & 0 \\ 0 & 0 & 1/r^2 & 0 \\ 0 & 0 & 0 & 1/r^2\sin^2\theta \end{pmatrix} \tag{8}$$

Therefore, energy-momentum tensor $T_{\mu\upsilon}$ is

$$T_{\mu\upsilon} = \frac{1}{4\pi c}(F_{\mu\rho}F_{\upsilon}{}^{\rho} - \frac{1}{4}g_{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma}) = \frac{1}{4\pi c}(g_{\mu\upsilon}F_{\upsilon\rho}F^{\upsilon\rho} - \frac{1}{4}g_{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma}) \tag{9}$$

Hence, if we calculate $T_{\mu\upsilon}$,

$$T_{\mu\upsilon} = \frac{E^2+B^2}{8\pi c}\begin{pmatrix} A & 0 & 0 & 0 \\ 0 & -B & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix} \tag{10}$$

Eq(5) is

$$\frac{\partial F^{\mu\upsilon}}{\partial x^{\upsilon}} + \Gamma^{\mu}{}_{\rho\upsilon}F^{\rho\upsilon} + \Gamma^{\upsilon}{}_{\rho\upsilon}F^{\mu\rho} = \frac{\partial F^{\mu\upsilon}}{\partial x^{\upsilon}} + \Gamma^{\upsilon}{}_{\rho\upsilon}F^{\mu\rho} \tag{11}$$

In this time,

$$\Gamma^{\upsilon}{}_{\rho\upsilon} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\rho}}\sqrt{-g} \tag{12}$$

Eq(5) is

$$F^{\mu\upsilon}{}_{;\upsilon} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\upsilon}}(\sqrt{-g}F^{\mu\upsilon}) = 0 \tag{13}$$

Hence, Eq(13) is

$$\frac{\partial}{\partial r}(r^2 E) = 0, \quad E = k\frac{Q}{r^2} \tag{14}$$

Eq(6) is

$$\frac{\partial F_{23}}{\partial r}+\frac{\partial F_{31}}{\partial x^2}+\frac{\partial F_{12}}{\partial x^3}=-\frac{\partial B}{\partial r}=0, B=0 \quad (15)$$

Hence, we know the following formula.

$$E=k\frac{Q}{r^2} \quad , \quad B=0 \quad (16)$$

The solution of Eq(4),Eq(5),Eq(6) is Reissner-Nodstrom solution. Hence,

$$A=1-\frac{C_1}{r}+\frac{C_2}{r^2} \quad , \quad B=1/(1-\frac{C_1}{r}+\frac{C_2}{r^2}) \quad (17)$$

In this time, if energy-momentum tensor $T^{\mu\upsilon}$ satisfy the energy-momentum conservation law (Noether's theorem)

$$\begin{aligned} T^{\mu\upsilon}{}_{;\upsilon} &= T^{00}{}_{;0}+T^{ii}{}_{i;}=T^{00}{}_{;0}+\frac{\partial T^{ii}}{\partial x^i}+2\Gamma^i{}_{ii}T^{ii} \\ &= \frac{E^2+B^2}{8\pi c}\frac{\partial(-g^{11})}{\partial r}+2\cdot\frac{1}{2}g^{11}(\frac{\partial g_{11}}{\partial r})\cdot\frac{E^2+B^2}{8\pi c}\cdot -g_{11} \\ &= \frac{E^2+B^2}{8\pi c}(-\frac{C_1}{r^2}+\frac{2C_2}{r^3})+\frac{E^2+B^2}{8\pi c}(\frac{C_1}{r^2}-\frac{2C_2}{r^3})=0 \end{aligned} \quad (18)$$

3. Conclusion

Reissner-Nodstrom solution hold by the energy-momentum density's conservation law of electromagnetic field in general relativity theory.

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