Riemann Hypothesis
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1 Abstract
The Proof involves Analytic Continuation of the Riemann Zeta function expressed as a functional equation.

Further as all the conditions of Rolle’s Theorem are satisfied, specific calculations produce the result.

Till date millions of non trivial zeroes of the Riemann Zeta function has been found on the critical line.

THE RIEMANN HYPOTHESIS (1859): states that the Riemann Zeta Function,

\[ \zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \]

has non trivial zeroes for \( \text{Re}(s) = 1/2 \).

2 Proof:
The Analytic Continuity of Riemann Zeta-function over

0 < Re(s) < 1

is defined as,

\[ \zeta^*(s) = \frac{(s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s))}{2}. \]
\( \zeta^*(0) = 0 \)
\( \zeta^*(1) = 0 \)

thus, \( \zeta^*(0) = \zeta^*(1) \)

Also, \( \zeta^* \) being Analytic is Continuous on \([0, 1]\) and Differentiable on \((0, 1)\)

So, By Rolle’s Theorem, there exists a \( s_0 \in (0, 1) \) such that,

\[
| \frac{d}{ds} \zeta^*(s_0) | = 0
\]
\[
| \frac{2\pi}{2} \pi^{-s_0/2} \Gamma(s_0/2) \zeta(s_0) | = 0
\]
\[
| (2s_0 - 1) | | \zeta(s_0) | = 0
\]
\[
| (2s_0 - 1) | = 0 \text{ or } | \zeta(s_0) | = 0
\]

\((2\sigma_0 - 1)^2 = 0, \sigma = \text{Re}(s)\)

implies, \( \sigma_0 = 1/2 \) or \( \zeta(1/2) = 0 \)

But, \( \zeta(1/2) = \sum_{n=1}^{\infty} 1/n^{1/2} = \infty \neq 0 \).

Therefore, \( \sigma = 1/2 \).

So, Real part = 1/2.

Let, imaginary part = \( t \).
so, we have \( s = 1/2 + it \)

then,

\[
| 2(1/2 + it) - 1\zeta(1/2 + it) | = 0
\]
\[
\zeta(1/2 + it) = 0
\]

Hence all the zeroes lie on the line \( x = 1/2 \) and are infinitely many.
3 References:-


2. Titchmarsh- The theory of Functions .


8. A note on S(t) and the zeros of the Riemann zeta-function - DA Goldston, SM Gonek.

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