Yes, P = NP, Says Calculus 1 & 2

Abstract

"5% of the people think; 10% of the people think that they think; and the other 85% would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

By applying differential and integral calculus, this paper covers the principles and procedures for producing the solution of a problem, given the procedure for checking the correctness of the solution of a problem, and vice versa. If one is able to check quickly and completely, the correctness of the solution of a problem, one should also be able to produce the solution of the problem by reversing the order of the steps of the checking process, while using opposite operations in each step. The above principles were applied to four examples from calculus as well as to an example from geometry. Even though in calculus, one normally uses differentiation to check the correctness of an integration result, one will differentiate a function first, and then integrate the derivative to obtain the original function. One will differentiate the trigonometric functions, tan *x,* cot *x,* sec *x* and csc *x;* followed by integrating each derivative to obtain each original function. The results show that the solution process and the checking process are inverses of each other. In checking the correctness of the solution of a problem, one should produce the complete checking procedure which includes the beginning, the middle, and the end of the problem. Checking only the correctness of the final answer or statement is incomplete checking. To facilitate complete checking, the question should always be posed such that one is compelled to show a complete checking procedure from which the solution procedure can be produced. A general application of P = NP is that, if the correctness of the solution of a problem can be checked quickly and it is difficult to write a solution procedure, then first, write a complete checking procedure and reverse the order of the steps while using opposite operations to obtain the solution of the problem. Therefore, P is equal to NP.

Introduction

Since differentiation and integration are inverse operations of each other, one will first differentiate as in Example 1a, and one will then reverse the steps in Example 1a, using inverse operations in each step as in Example 1b. That is, one will replace the differentiation symbols by integration symbols. In reversing the steps, the last step of Example 1a becomes the first step of Example 1b; and the first step of Example 1a becomes the last step of Example 1b.

P Differentiation

Example 1a: Given that $y = \tan x$, **(1)**

Show that $\frac{dy}{dx} = \sec^2 x$ **(2)**

Solution

$$y = \frac{\sin x}{\cos x} \quad (\tan x = \frac{\sin x}{\cos x}) \qquad \textbf{(3)}$$

$$\frac{dy}{dx} = \frac{\cos x \frac{d}{dx}[\sin x] - \sin x \frac{d}{dx}[\cos x]}{\cos^2 x} \qquad \textbf{(4)}$$

(Using the quotient rule)

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} \qquad \textbf{(5)}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \qquad \textbf{(6)}$$

$$= \frac{1}{\cos^2 x} \quad (\cos^2 x + \sin^2 x = 1) \qquad \textbf{(7)}$$

$$= \left(\frac{1}{\cos x}\right)^2 \qquad \textbf{(8)}$$

$$= \sec^2 x \quad (\frac{1}{\cos x} = \sec x) \qquad \textbf{(9)}$$

$$\therefore \frac{dy}{dx} = \sec^2 x \qquad \textbf{(10)}$$

Scrapwork for the column on the right-->

$$\boxed{\int \frac{\sin x}{\cos^2 x} dx \Rightarrow}.$$

Apply u-substitution

Let $s = \cos x$

$$\frac{ds}{dx} = -\sin x \text{ and } dx = \frac{ds}{-\sin x}$$

$$\int \frac{\sin x}{\cos^2 x} dx = \int \frac{\sin x}{s^2} \bullet \frac{ds}{-\sin x}$$

$$= -\int \frac{ds}{s^2}$$

$$= -\int s^{-2} ds = -\frac{s^{-1}}{-1} = \frac{1}{s}$$

$$\boxed{\therefore \int \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x}}$$

NP Integration (See also Example 5)

Example 1b: Given that $\frac{dy}{dx} = \sec^2 x$,

Show that $y = \tan x$..

Solution $y = \int \sec^2 x dx$ **(1)**

$$= \int \left(\frac{1}{\cos x}\right)^2 dx \quad (\frac{1}{\cos x} = \sec x\ ) \qquad \textbf{(2)}$$

$$= \int \frac{1}{\cos^2 x} dx \quad () \qquad \textbf{(3)}$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x} dx \quad (\sin^2 + \cos^2 x = 1\ ) \qquad \textbf{(4)}$$

$$= \int \frac{\sin^2 x}{\cos^2 x} dx + \int \frac{\cos^2 x}{\cos^2 x} dx \qquad \textbf{(5)}$$

$$= \int \frac{\sin^2 x}{\cos^2 x} dx + \int 1 dx \qquad \textbf{(7)}$$

$$= \int \frac{\sin x(\sin x)}{\cos^2 x} dx + \int 1 dx \qquad \textbf{(8)}$$

(Integrate the first integral by parts)

Let $u = \sin x$; $dv = \frac{\sin x}{\cos^2 x} dx$ (version1) or

$$v = \frac{\sin x}{\cos^2 x} \quad \text{(version 2)} \qquad \textbf{(9)}$$

$$y = \sin x \int \frac{\sin x}{\cos^2 x} - \int [\frac{d}{dx}(\sin x) \int \frac{\sin x}{\cos^2 x}] dx \qquad \textbf{(10)}$$

$$= \sin x(\frac{1}{\cos x}) - \int [(\cos x)(\frac{1}{\cos x})] dx \qquad \textbf{(11)}$$

$$= \frac{\sin x}{\cos x} - \int 1 dx + \int 1\ dx \qquad \textbf{(12)}$$

$$= \frac{\sin x}{\cos x} + c \qquad \textbf{(13)}$$

$$\therefore y = \tan x + c \qquad \textbf{(14)}$$

Note above that equation **10** (the last equation) in Example 1a with the differentiation symbol replaced by the integration symbol becomes equation **(1)** of Example 1b.; and the first equation in Example 1a becomes the last equation of Example 1b

.**Note**: $\int (\frac{d}{dx}(\tan x)) dx = \int \sec^2 x dx$

P Differentiation

Example 2a: Given that $y = \cot x$, **(1)**

Show that $\frac{dy}{dx} = -\csc^2 x$.

Solution

$$y = \frac{\cos x}{\sin x} \quad (\cot x = \frac{\cos x}{\sin x})$$

$$\frac{dy}{dx} = \frac{\sin x \frac{d}{dx}[\cos x] - \cos x \frac{d}{dx}[\sin x]}{\sin^2 x} \quad \textbf{(2)}$$

(Using the quotient rule)

$$= \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x} \quad \textbf{(3)}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} \quad \textbf{(4)}$$

$$= \frac{-[\sin^2 x + \cos^2 x]}{\sin^2 x}$$

$$= -\frac{1}{\sin^2 x} \quad (\sin^2 x + \cos^2 x = 1) \quad \textbf{(5)}$$

$$= -\left(\frac{1}{\sin x}\right)^2 \quad (\frac{1}{\sin x} = \csc x) \quad \textbf{(6)}$$

$$= -\csc^2 x \quad \textbf{(7)}$$

$$\therefore \frac{dy}{dx} = -\csc^2 x.$$

Integration scrap work for $\boxed{\int \frac{\cos x}{\sin^2 x} dx}$

Integrate by u-substitution

Let $w = \sin x$.**Then** $\frac{dw}{dx} = \cos x$;

$$dx = \frac{dw}{\cos x}$$

$$\int \frac{\cos x}{\sin^2 x} dx = \int \frac{\cos x}{w^2} \bullet \frac{dw}{\cos x}$$

$$= \int \frac{dw}{w^2} = \int w^{-2} dw$$

$$= -\frac{1}{w} = \boxed{-\frac{1}{\sin x}}$$

$$\int \frac{\cos x}{\sin^2 x} dx \boxed{= -\frac{1}{\sin x}}$$

NP Integration (See also Example 6)

Example 2b: Given that $\frac{dy}{dx} = -\csc^2 x$,

Show that $y = \cot x$

Solution

$$y = \int -\csc^2 x dx$$

$$= -\int \left(\frac{1}{\sin x}\right)^2 dx \quad (\frac{1}{\sin x} = \csc x)$$

$$= -\int \frac{1}{\sin^2 x} dx$$

$$= \int \frac{-[\sin^2 x + \cos^2 x]}{\sin^2 x} dx \quad (\sin^2 x + \cos^2 x = 1)$$

$$= \int \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} dx$$

$$= \int -\frac{\sin^2 x}{\sin^2 x} dx + \int \frac{-\cos x(\cos x)}{\sin^2 x} dx$$

$$= -\int 1 dx + \int \frac{-\cos x(\cos x)}{\sin^2 x} dx$$

$$= \int \frac{-\cos x(\cos x)}{\sin^2 x} dx - \int 1 dx$$

(Integrate first integral by parts.)

Let $u = -\cos x$; $dv = \frac{\cos x}{\sin^2 x} dx$ (version1) or

$v = \frac{\cos x}{\sin^2 x}$ (version 2)

$$y = -\cos x \int \frac{\cos x}{\sin^2 x} dx - \int [\frac{d}{dx}(-\cos x) \int \frac{\cos x}{\sin^2 x}] dx - \int 1 dx$$

$$= -\cos x(-\frac{1}{\sin x}) - \int [(\sin x)(-\frac{1}{\sin x})] dx - \int 1 dx$$

$$= \frac{\cos x}{\sin x} - \int -1 dx - \int 1 dx$$

$$= \cot x + \int 1 dx - \int 1 dx$$

$$= \cot x + x - x + c$$

$$= \cot x + c$$

$$y = \int -\csc^2 x dx$$

$$y = -\int \csc^2 x dx = \cot x + C$$

Also, $\int \csc^2 x dx = -\cot x + C$

Note above that one could have integrated equation (3) of Example 2a instead of equation ((4) of Example 2a.

P **Differentiation**

Example 3a: Given that $y = \sec x$,

Show that $\frac{dy}{dx} = \sec x \tan x$.

Solution (We apply the chain rule)

$$y = \frac{1}{\cos x} \qquad (\sec x = \frac{1}{\cos x})$$

$$y = (\cos x)^{-1}$$

Let $u = \cos x$

Then $y = u^{-1}$

$$\frac{dy}{du} = -u^{-2}$$

$$= -\frac{1}{u^2}$$

$$= -\frac{1}{\cos^2 x} \qquad (u = \cos x)$$

$$\frac{du}{dx} = -\sin x \qquad (u = \cos x)$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \quad \text{(Chain rule)}$$

$$\frac{dy}{dx} = (-\frac{1}{\cos^2 x})(-\sin x)$$

$$\frac{dy}{dx} = \frac{\sin x}{\cos^2 x} \qquad \textbf{(1)}$$

$$\frac{dy}{dx} = \frac{\sin x}{\cos x} \bullet \frac{1}{\cos x}$$

$$\frac{dy}{dx} = \tan x \bullet \sec x \qquad \textbf{(2)}$$

$$\frac{dy}{dx} = \sec x \tan x. \qquad \textbf{(3)}$$

NP **Integration**

Example 3b: Given that $\frac{dy}{dx} = \sec x \tan x$,

Show that $y = \sec x$

Solution

$$y = \int \tan x \bullet \sec x dx.$$

$$= \int \frac{\sin x}{\cos x} \bullet \frac{1}{\cos x} dx$$

$$= \int \frac{\sin x}{\cos^2 x} dx \quad \text{(integrate by u-substitution)}$$

Let $u = \cos x$. Then $\frac{du}{dx} = -\sin x$, and $dx = -\frac{du}{\sin x}$

$$y = \int \frac{\sin x}{\cos^2 x}$$

$$= \int \frac{\sin x}{u^2} \bullet (-\frac{du}{\sin x})$$

$$= -\int \frac{1}{u^2} du$$

$$= -\int u^{-2} du$$

$$= -(-u^{-1})$$

$$= \frac{1}{u} + C$$

$$= \frac{1}{\cos x} + C \qquad \textbf{(1)}$$

$$y = \sec x + C \qquad \textbf{(2)}$$

Solution B

$$y = \int \tan x \bullet \sec x dx$$

Integrate by u-substitution

Let $u = \sec x$ Then $\frac{du}{dx} = \sec x \tan x$ and

$dx = \frac{du}{\sec x \tan x}$. Substituting for u and dx,

$y = \int \tan x \bullet \sec x dx$ becomes

$$= \int \frac{\tan x \bullet u du}{u \tan x}$$

$$= \int u du$$

$$= u + c$$

$$\therefore y = \sec x + c$$

With regards to $\sec^2 x$, $\csc^2 x$, $\sec x$ $\sec x \tan x$, $\csc x \cot x$ there is more than one approach to obtain the integrals of these functions.

P — Differentiation

Example 4a: Given that $y = \csc x$,

Show that $\frac{dy}{dx} = -\csc x \cot x$

.**Solution** (We apply the chain rule)

$$y = \frac{1}{\sin x} \qquad (\csc x = \frac{1}{\sin x})$$

$$y = (\sin x)^{-1}$$

Let $u = \sin x$

Then $y = u^{-1}$

$$\frac{dy}{du} = -u^{-2}$$

$$= -\frac{1}{u^2}$$

$$= -\frac{1}{\sin^2 x} \quad (u = \sin x)$$

$$\frac{du}{dx} = \cos x \qquad (u = \sin x)$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \quad \text{(Chain rule)}$$

$$\frac{dy}{dx} = (-\frac{1}{\sin^2 x})(\cos x)$$

$$\frac{dy}{dx} = -\frac{\cos x}{\sin^2 x} \qquad \textbf{(1)}$$

$$\frac{dy}{dx} = -\frac{\cos x}{\sin x} \bullet \frac{1}{\sin x}$$

$$\therefore \frac{dy}{dx} = -\cot x \bullet \csc x \qquad \textbf{(2)}$$

Note above that if we were asked to find $\frac{dy}{dx}$**,** we could leave the answer as in equation (1) instead of as in equation (2). However, the wording of the question compelled us to end as in equation (2)..

NP — Integration

Example 4b: Given that $\frac{dy}{dx} = -\csc x \cot x$,

Show that $y = \csc x + c$

Solution A

$$y = \int -\cot x \bullet \csc x dx \qquad \textbf{(1)}$$

$$= -\int \frac{\cos x}{\sin x} \bullet \frac{1}{\sin x} dx$$

$$= -\int (\frac{1}{\sin^2 x})(\cos x) dx$$

$$= -\int \frac{\cos x}{\sin^2 x} dx \quad \text{Integrate by u-substitution}$$

Let $u = \sin x$. Then $\frac{du}{dx} = \cos x$, and $dx = \frac{du}{\cos x}$

$$-\int \frac{\cos x}{\sin^2 x} dx = -\int \frac{\cos x}{u^2} \frac{du}{\cos x}$$

$$= -\int \frac{du}{u^2}$$

$$= -\int u^{-2} du = \frac{-u^{=1}}{-1} = -(-\frac{1}{u})$$

$$= \frac{1}{u}$$

$$= \frac{1}{\sin x} + c \qquad \textbf{(2)}$$

$$y = \csc x + c$$

$$y = -\int \cot x \bullet \csc x dx = \csc x + c$$

Also, $\int \cot x \bullet \csc x dx = -\csc x + c$ **(3)**

Solution B

Apply u-substitution to equation (1) above

$$y = \int -\cot x \bullet \csc x dx$$

Let $u = \csc x$ Then $\frac{du}{dx} = -\csc x \cot x$ and

$dx = -\frac{du}{\csc x \cot x}$**.** Substituting for u and dx

$y = \int -\cot x \bullet \csc x dx$ becomes

$$= \int \frac{-\cot x \bullet u du}{-u \cot x}$$

$$= \int u du$$

$$= u + c$$

$$= \csc x + c$$

$$y = \int -\cot x \bullet \csc x dx = \csc x + c$$

Also, $\int \cot x \bullet \csc x dx = -\csc x + c$

From the above examples, one could generalize that if the derivative one wants to integrate was obtained by the quotient or product rule, then one will use integration-by-parts to integrate; but if the derivative one wants to integrate was obtained by the chain rule, then one will use u-substitution to integrate.

Straightforward Integration of Examples 1b and 2b

Example 5 Find $\int \sec^2 x dx$

$\int \sec^2 x dx = \int \tan^2 x dx + \int 1 \, \mathrm{dx}$ (Apply Trig identity) (1)

$$= \int \frac{\sin^2 x}{\cos^2 x} dx + \int 1 \, dx \quad (2)$$

$$= \int \frac{\sin x(\sin x)}{\cos^2 x} dx + \int 1 \, dx \quad (3)$$

Let $u = \sin x$; $v = \dfrac{\sin x}{\cos^2 x}$

$$= \int \frac{\sin x(\sin x)}{\cos^2 x} dx + \int 1 \, dx \quad (4)$$

$$= \sin x \int \frac{\sin x}{\cos^2 x} - \int \left[\frac{d}{dx}(\sin x) \int \frac{\sin x}{\cos^2 x}\right] dx + \int 1 \, dx \quad (5)$$

$$= \sin x\left(\frac{1}{\cos x}\right) - \int \left[(\cos x)\left(\frac{1}{\cos x}\right)\right] dx + \int 1 \, dx \quad (6)$$

$$= \frac{\sin x}{\cos x} - \int 1 dx + \int 1 \, dx \quad (7)$$

$$= \tan x - x + x + c \quad (8)$$

$$= \tan x + c$$

Scrapwork

$$\boxed{\int \frac{\sin x}{\cos^2 x}}$$

Let $s = \cos x$

$\dfrac{ds}{dx} = -\sin x$ and $dx = -\dfrac{ds}{\sin x}$

$$\int \frac{\sin x}{\cos^2 x} = \int \frac{\sin x}{s^2} \bullet \left(-\frac{ds}{\sin x}\right)$$

$$= -\int \frac{ds}{s^2}$$

$$= -\int s^{-2} ds = -\frac{s^{-2+1}}{-1}$$

$$= \frac{s^{-1}}{1} = \frac{1}{s} = \frac{1}{\cos x}$$

$$\boxed{\therefore \int \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x}}$$

.

Example 6

Find $\int \csc^2 x dx$

$$\int \csc^2 x dx = \int \cot^2 x dx + \int 1 \, \mathrm{dx} \quad (1)$$

$$= \int \frac{\cos^2 x}{\sin^2 x} dx + \int 1 \, dx \quad (2)$$

$$= \int \frac{\cos x(\cos x)}{\sin^2 x} dx + \int 1 \, dx \quad (3)$$

Let $u = \cos x$; $v = \dfrac{\cos x}{\sin^2 x}$

$$= \int \frac{\cos x(\cos x)}{\sin^2 x} dx + \int 1 \, dx \quad (4)$$

$$= \cos x\left(-\frac{1}{\sin x}\right) - \int \left[\frac{d}{dx}(\cos x)\left(-\frac{1}{\sin x}\right)\right] dx + \int 1 \, dx \quad (5)$$

$$= \cos x\left(-\frac{1}{\sin x}\right) - \int \left[(-\sin x)\left(-\frac{1}{\sin x}\right)\right] dx + \int 1 \, dx \quad (6)$$

$$= -\frac{\cos x}{\sin x} - \int 1 dx + \int 1 \, dx \quad (7)$$

$$= -\cot x - x + x + c \quad (8)$$

$$= -\cot x + c$$

Scrapwork

$$\boxed{\int \frac{\cos x}{\sin^2 x}}$$

Let $s = \sin x$

$\dfrac{ds}{dx} = \cos x$ and $dx = \dfrac{ds}{\cos x}$

$$\int \frac{\cos x}{\sin^2 x} = \int \frac{\cos x}{s^2} \bullet \frac{ds}{\cos x}$$

$$= \int \frac{ds}{s^2}$$

$$= \int s^{-2} ds$$

$$= \frac{s^{-2+1}}{-1}$$

$$= -\frac{s^{-1}}{1}$$

$$= -\frac{1}{s}$$

$$\boxed{\therefore \int \frac{\cos x}{\sin^2 x} = -\frac{1}{\sin x}}$$

Geometric Example
Proof

If $\frac{a}{b} = \frac{c}{d}$, show that $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

Plan: 1. From conclusion to hypothesis; followed by

2. Reversing the steps while using opposite operations to obtain the conclusion

P: From **conclusion** to **hypothesis**	**NP : Proof** (From **hypothesis** to **conclusion**) Reverse the steps in **A** (on left) to obtain the proof
1. $\frac{a-b}{a+b} = \frac{c-d}{c+d}$	1. $\frac{a}{b} = \frac{c}{d}$
2. $(a-b)(c+d) = (a+b)(c-d)$	2. $ad = bc$
3. $ac + ad - bc - bd = ac - ad + bc - bd$	3. $2ad = 2bc$
4. $ad - bc = -ad + bc$	4. $ad - bc = -ad + bc$
5. $2ad = 2bc$	5. $ac + ad - bc - bd = ac - ad + bc - bd$
6. $ad = bc$	6. $(a-b)(c+d) = (a+b)(c-d)$
7. $\frac{a}{b} = \frac{c}{d}$	7. $\frac{a-b}{a+b} = \frac{c-d}{c+d}$ Q.E.D.

Observe above that Step 7 of **P** becomes Step 1 of **NP;** Step 1 of **P** becomes Step 7 of **NP**

NP Expanded

$\frac{a}{b} = \frac{c}{d}$

$ad = bc$

$2ad = 2bc$

$ad + ad = bc + bc$

$ac + ad + ad = ac + bc + bc$ (add ac to both sides)

$ac + ad - bc = ac - ad + bc$ (subtract ad from both sides and subtract bc from both sides)

$ac + ad - bc - bd = ac - ad + bc - bd$ (subtract bd from both sides)

$a(c+d) - b(c+d) = a(c-d)) + b((c-d)$ (factoring by grouping)

$(a-b)(c+d) = (a+b)((c-d)$

$\frac{a-b}{a+b} = \frac{c-d}{c+d}$

Discussion

Normally. we use differentiation to check the correctness of integration results. However, we can also use integration to check the correctness of differentiation results, even though differentiation is usually relatively easier than integration; and it would be a good challenge. In using some of the free on-line integral calculators, when asked to integrate $\sec^2 x$ the response was $\tan x$, without showing any steps used in arriving at this correct answer. In a number of textbooks, the response is that, " since the derivative of $\tan x$ is $\sec^2 x$, the integral of $\sec^2 x$ is $\tan x$" with no steps as to how to obtain $\tan x$ from $\sec^2 x$. Such responses imply that answers can be produced but detailed integration steps may not be produced when using a computer. $Sec^2 x$ can easily be integrated by hand using integration-by- parts. Similar responses from above were obtained for the differentiation of $\cot x$. Computers should be programmed in such a way that it is able to provide the steps from the beginning of the problem to the answer. If the correctness of the solution to a problem is easy to check, should the problem be easy to solve? The answer is in the affirmative. However, it is important to communicate properly for desired response. In the above examples, note how the questions were posed. For example, in Example 1b, the question was posed as "Show that $y = \tan x$." . If the question had been "find the integral of $\sec^2 x$", one could answer the question as "because the derivative of $\tan x$ is $\sec^2 x$, the integral of $\sec^2 x$ is $\tan x$"; and in this case, one would not be compelled to show the detailed steps from $\sec^2 x$ to $\tan x$ as in Example 1b.

Conclusion

Completely solving and completely checking the solutions of mathematical problems are inverse processes of each other. If the checking process is easy, the solution process should also be relatively easy, and vice versa. Computer hardware and software should be designed in such a way that the checking process is complete. Just checking answers is incomplete checking. For practical purposes, checking answers only is sometimes insufficient. Even, if it is difficult to check, it would not be difficult to solve, since the complete checking process provides sufficient information to reverse to obtain the solution. Thus, if the correctness of a problem can be checked in polynomial time, the problem can be solved in polynomial time. The question should always be posed such that one is compelled to show a checking procedure from which the solution procedure can be deduced. A general application of P = NP is that, if the correctness of the solution of a problem can be checked quickly and it is difficult to write a solution procedure, then first, write a complete checking procedure and reverse the order of the steps of the checking procedure while using opposite operations in each step, to obtain the solution procedure for the problem. Therefore, P is equal to NP.

Adonten

EXTRA
A: Calculus Humor

1a, **Question**: Mr. $\tan x$, what did you use to change your appearance to $\sec^2 x$?.
Answer: I used the quotient or product rule
1b. **Question**: What rule would you use to change your appearance back to $\tan x$, and why?.
Answer: I will use integration--by-parts because integration by parts will "undo" or reverse what the product or quotient rule did.

2a. **Question**: Mr. $\cot x$, what rule did you use to change your appearance to $-\csc^2 x$?.
Answer: I used the quotient or product rule.
2b. **Question**: What rule would you use to change your appearance back to $\cot x$, and why?.
Answer: I will use integration--by-parts because integration by parts will "undo" or reverse what the product or quotient rule did.
3a, **Question**: Mr. $\sec x$, what rule did you use to change your appearance to $\sec x \tan x$.
Answer: I used the chain rule for differentiation.
3b. **Question**: What rule would you use to change your appearance back to $\sec x$, and why?.
Answer: I will use integration by u-substitution, because integration by u-substitution will undo or reverse what the chain rule did.
4a, **Question**: Mr. $\csc x$, what rule did you use to change your appearance to $-\csc x \cot x$.
Answer: I used the chain rule for differentiation.
4b. **Question**: What rule would you use to change your appearance back to $\csc x$, and why?.
Answer: I will use integration by u-substitution, because integration by u-substitution will undo or reverse what the chain rule did.

5a. **Question**: Mr. $\frac{1}{3}x^3$, what rule did you use to change your appearance to x^2.
Answer: I used the power rule for differentiation.

5b. **Question**: What rule would you use to change your appearance back to $\frac{1}{3}x^3$, and why?.
Answer: I will use the power rule for integration because the power rule for integration will undo or reverse what the power rule for differentiation did.

B: Terminology Consistency

Existing Terminology

Differentiation	Integration
Product rule	Integration–by–parts
Chain rule	U–substitution

Some Suggested Terminology

Differentiation		Integration
Product rule for differentiation		Product rule for integration
	or	
Differentiation–by–parts		Integration–by–parts
Chain rule for differentiation		Chain rule for integration
	or	
U–substitution for differentiation		U–substitution for integration
Power rule for differentiation		Power rule for integration (exists already)

For more on U-substitution and Integration-by-parts, see pages 210 & 244 respectively, of Calculus 1 & 2 by A. A. Frempong, published by Yellowtextbooks.com

C: Programmer & Computer Conversation

Programmer: Mr. Computer , kindly integrate $\sec^2 x$ for me..

Mr. Computer: The integral of $\sec^2 x$ is $\tan x$, because the derivative of $\tan x$ is $\sec^2 x$

Programmer: You did not give the steps used in going from $\sec^2 x$ to $\tan x$

Mr. Computer: You did not ask for the steps in integrating $\sec^2 x$, You asked for the integral of $\sec^2 x$ and the correct answer is $\tan x$ because the derivative of $\tan x$ is $\sec^2 x$. If you check some of the calculus textbooks for the integral of $\sec^2 x$,they will say that the integral of $\sec^2 x$ is $\tan x$ because the derivative of $\tan x$ is $\sec^2 x$; and they would not show the detailed steps. If you want all the steps involved in going from $\sec^2 x$ to $\tan x$ as in the solution in Example 1b, above, you must ask so, but before doing so, you should provide my system with instructions which will allow me to give a detailed solution as in Example 1b

Programmer: Thank you, Mr. Computer. I am going to give your system provisions that will allow you to integrate as in Example 1b. Also, if you can provide the complete differentiation as in Example 1a, I will be able to obtain the integration as in Example 1b.

Mr. Computer: I would appreciate the precise instruction for obtaining the complete integration of $\sec^2 x$, similar to the integration in Example 1b. Try to spend 1-3 weeks and you would produce excellent instructions. I read about complaints that I can quickly check the correctness of the solution of a problem but I am unable to provide the solution steps quickly. This not true. If you asked someone for his or her name, and he/she tells you his or her first name, it would be wrong to complain that the person did not give his or her full name. If you want the full name, you must ask for the full name. I have tables of integrals and derivatives of various possibilities in my system. From these tables, I can answer questions without showing the steps or all the steps. I you want all the steps, the question must ask for all the steps, otherwise, for efficiency, I will provide the shortest answer.

Programmer: I am going to provide your system with what you need, and I will also ask questions without ambiguity so that you provide the complete solutions. Thank you, Honorable Computer, for your service for the past 80 years

Mr. Computer: You are welcome (or Don't mention)

Adonten