

# On Newtonian dynamics with a variable Earth mass: Geodetic evidence and its implications on Pioneer spacecraft anomaly and LAGEOS satellite

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\*This paper is dedicated to Prof. Jayant Narlikar for developing VMH cosmology.

## Abstract

The Earth expansion problem has attracted great interest, and recent study gives geodetic evidence that the Earth has been expanding, at least over the recent several decades. In the meantime, Jayant Narlikar & Halton Arp have argued on possible variable mass hypothesis cosmology (VMH). Therefore, in the present article discusses some interesting effects related to varying  $G$ , but here we argue that instead of varying  $G$  we can think of varying mass ( $M$ ). Among other things we discuss receding planets from the Sun, calculation of Pioneer spacecraft anomaly as proposed by B.G. Sidharth, and also possible slight change in LAGEOS Satellite. Further observation to verify or refute this conjecture is recommended, especially based on geological evidence of expanding earth.

*Keywords:* Varying  $G$ , varying  $M$ , Pioneer spacecraft anomaly, Sidharth, gravitation, LAGEOS

## Introduction

The Earth expansion problem has attracted great interest, and recent study gives geodetic evidence that the Earth has been expanding, at least over the recent several decades. In the meantime, Jayant Narlikar & Halton Arp have argued on possible variable mass hypothesis cosmology (VMH). [44][45]

In 1974 Hoyle and Narlikar introduced a new type of theory of gravitation, based on Mach's principle, which Narlikar, Das and Arp have advanced further. Over the past 3 to 4 decades a large body of observational evidence has been gathered that points to the possibility that high-redshift quasars are physically associated with low-redshift galaxies. The excess (or anomalous) redshifts of these quasars are unlikely to be either of Doppler or of gravitational origin. Narlikar and Das suggested a new source for this excess redshift, resulting from the accumulation of inertial mass of a newly ejected particle by the ever expanding sphere of gravitational influence of the surrounding matter field. Narlikar and Das have shown that observed quasar alignments, and redshift bunching around preferred values can be understood within the framework of this new theory. For a detailed discussion of the variable mass hypothesis see section III of reference [44]. Narlikar and Arp describe a cosmology that is equivalent to the standard F-L cosmology with space curvature constant  $k = 0$ , i.e. Euclidean space.[45]

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So, what is new here? This paper describe a varying Mass solar system and earth dynamics, and it starts with a very simple assumption of Bose-Einstein condensate processes behind mass creation inside Earth inner core, although the actual process which takes place remains mysterious for now.

It is known, that the use of Bohr radius formula to predict celestial quantization has led to numerous verified observations [1]. This approach was based on Bohr-Sommerfeld quantization rules [2][3]. While this kind of approach is not widely accepted yet, this could be related to wave mechanics equation to describe large-scale structure of the Universe [4], and also a recent suggestion to reconsider Sommerfeld's conjectures in Quantum Mechanics [5]. Some implications of this quantum-like approach include exoplanet prediction, which becomes a rapidly developing subject in recent years [6][7].

Rubicic & Rubicic's approach [2] is particularly interesting in this regard, because they begin with a conjecture that Planck mass ( $m_p = \sqrt{\hbar c / 2\pi G}$ ) is the basic entity of Nature, which apparently corresponds to Winterberg's assertion of superfluid Planckian aether comprised of phonon-roton pairs [8]. In each of these pairs, superfluid vortices can form with circulation quantized according to  $\oint v_{\pm} \cdot dx = n\hbar / m_p$ . This condition implies the Helmholtz vortex theorem,  $d / dt \oint v_{\pm} \cdot dx = 0$ . This relationship seems conceivable, at least from the viewpoint of likely neat linkage between cosmology phenomena and various low-temperature condensed matter physics [9][10][11]. In effect, celestial objects at various scales could be regarded as spinning Bose-Einstein condensate; which method has been used for neutron stars [32].

Despite these aforementioned advantages, it is also known that all of the existing celestial quantization methods [1][2][3] thus far have similarity that they assume a circular motion, while the actual celestial orbits (and also molecular orbits) are elliptical. Historically, this was the basis of Sommerfeld's argument in contrast to Bohr's model, which also first suggested that any excess gravitational-type force would induce a precessed orbit [12]. In other words, any extra gravitational force will exert anomalous acceleration. We consider this effect can be observed not only in Earth (caused by expanding Earth), but also spiralling planetary orbits outward of the sun (receding planets), and in the last section we will discuss possible effect of varying M in Pioneer spacecraft anomaly.

## Prediction of the receding planets from the Sun

Now let suppose that the concept of varying Mass (increasing mass of the Sun) will cause the motion of planets around the Sun takes the shape of spiral instead of circle or elliptical.

In this regard, it is interesting to note that Sidharth has argued in favor of varying G [21]. From this starting point, he was able to explain –among other things-- anomalous precession (Lense-Thirring effect) of the first planet and also anomalous Pioneer acceleration. This will be discussed in the subsequent section. In principle, Sidharth's basic assertion is [21]:

$$G = G_{\otimes} \cdot (1 + t / t_{\otimes}) \quad (1)$$

It is worth noting here that Barrow [40c] has also considered a somewhat similar argument in the context of varying constants:

$$G = G_{\otimes} t_{\otimes} / (t - c) \quad (1a)$$

However, in this article we will use (1) instead of (1a), partly because it will lead to more consistent predictions with observation data. Alternatively, we could also hypothesize using Maclaurin formula:

$$G = G_{\otimes} e^{t/t_{\otimes}} = G_{\otimes} (1 + t/t_{\otimes} + (t/t_{\otimes})^2/2! + (t/t_{\otimes})^3/3! + \dots) \quad (1b)$$

Therefore, from this viewpoint equation (1) could be viewed as first-order approximation of (1b), by neglecting second and higher orders in the series. It will be shown in subsequent sections, that equation (1) is more convenient for deriving predictions.

If we conjecture that instead of varying  $G$ , the spinning mass  $M$  varies, then it would result in the same effect as explained by Sidharth [21], because for Keplerian dynamics we could assert  $k=GM$ , where  $k$  represents the stiffness coefficient of the system. Accordingly, Gibson [22] has derived similar conjecture of exponential mass flux from Navier-Stokes gravitational equation, which can be rewritten in the form:

$$M = G_{\otimes} e^{t/t_{\otimes}} = M_{\otimes} (1 + t/t_{\otimes} + (t/t_{\otimes})^2/2! + (t/t_{\otimes})^3/3! + \dots) \quad (1c)$$

provided we denote for consistency [22]:

$$t_{\otimes} = \tau_g / 2\pi \quad (1d)$$

Using the above argument of Maclaurin series, equation (1c) could be rewritten in the similar form with (1) by neglecting higher order effects:

$$M = M_{\otimes} (1 + t/t_{\otimes}) \quad (2)$$

In a recent article Gibson & Schild [23] argue that their gravitational Navier-Stokes approach results in better explanation than what is offered by Jeans instability. Furthermore, R.M. Kiehn has also shown that the Navier-Stokes equation corresponds exactly to Schroedinger equation [27], which seems to support the idea of quantization of celestial motion [1][2][3]. A plausible extension of Euler equation and Jeans instability to describe gravitational clustering has been discussed in [22b], which corresponds to viscosity term and also turbulence phenomena [22c,22d] described by Gibson. Therefore, apparently equation (10) is more consistent with kinematical gravitational instability consideration than (9).

From equation (2) we could write for  $M$  at time difference  $\Delta t = t_2 - t_1$ :

$$M_2 = M_{\otimes} (1 + t_2/t_{\otimes}) \quad (3)$$

$$M_1 = M_{\otimes} (1 + t_1/t_{\otimes}) \quad (4)$$

from which we get:

$$\Delta M = (M_{\otimes} / t_{\otimes}) (t_2 - t_1) \quad (5)$$

Inserting our definition  $\Delta t = t_2 - t_1$  yields:

$$\Delta M / \Delta t = (M_{\otimes} / t_{\otimes}) = k \quad (6)$$

For verification of this assertion, we could use equation (6) to predict mass flux of the Earth.

Inserting equation (14) into equation (5a), we get:

$$M_{\otimes} / t_{\otimes} \approx -\dot{\omega}.MR^2\omega / (5.c^2) \quad (7)$$

which is the basic conjecture of the present article. From this viewpoint we could rewrite equation (8j) and (8k):

$$dr / dt = M / \chi.[1 - \dot{\omega}.R^2\omega / (5.c_s^2)] - r \quad (7a)$$

and inserting equation (15), we get:

$$dr / dt = M / \chi.[1 + M_{\otimes} / (t_{\otimes}.M)] - r \quad (7b)$$

A plausible test of this conjecture could be made by inserting this result (14) into equation (8e) and using  $M_{\otimes} = 1.9895 \times 10^{33} \text{ g}$  and  $t_{\otimes} = 2 \times 10^{10} \text{ year}$  as the epoch of the solar system [21], and specific velocity  $v_0 = 144 \text{ km/sec}$  [1], then from equation (7b) we get a receding orbit radius for Earth at the order of:

$$\Delta r_{\text{Earth}} / \Delta t = 6.03 \text{ m / year} \quad (8)$$

Interestingly, there is an article [24] hypothesizing that there is a tad effect of *receding Earth orbit from the Sun at the order of 7.5 m/year*, supposing Earth orbit radius has been expanding as large as  $93 \times 10^6$  miles since the beginning of the solar epoch at  $t_{\otimes} = 2 \times 10^{10} \text{ year}$  ago (in the quoted article, it was assumed that the epoch is  $4.5 \times 10^9$  years). Of course, it shall be noted that there is large uncertainty of the estimate of solar epoch, for instance Gibson prefers  $4.6 \times 10^{17} \text{ sec}$  (or  $1.46 \times 10^{10} \text{ year}$ , see [22]). Therefore, it is suggested here to verify this assumption of solar epoch using the same tad effect for other planets. For observation purposes, some estimate values were presented in Table 2 using the same approach.

Table 2. Prediction of planetary orbit radii (r) increment

Celestial object	Quantum number (n)	Orbit increment (m/yr)
Mercury	3	2.17
Venus	4	3.86
Earth	5	6.03
Mars	6	8.68

## Remark on Pioneer spacecraft anomaly

As we pointed out above, Sidharth has come up with a simple method to account for Pioneer 10 anomalous acceleration, by deducing from well-known Keplerian equation and  $G = G_{\otimes} \cdot (1 + t / t_{\otimes})$ , then he obtains  $a = 10^{-11} \text{ cm/sec}^2$  [21]. It is very interesting to note here, that Sidharth's method was based on assumption varying G constant, but he also obtains:

$$rv^2 \approx GM$$

Therefore, actually this calculation will be much similar for explaining Pioneer 10 anomalous acceleration if we assume varying M

$$M = M_{\otimes} \cdot (1 + t / t_{\otimes}),$$

Instead of varying G (equation 1). In other words, varying M hypothesis has no effect on calculation of Pioneer anomalous acceleration, but it has more substantial meaning, i.e. an effect of gravitational Navier-Stokes (see Gibson [22]).

Other studies on Pioneer spacecraft anomaly have been reported, but it seems to us that Sidharth's method is one of the simplest explanation and most consistent to cosmology or Dirac's Large number hypothesis.

## Geodetic evidence of Expanding Earth and a plausible test using LAGEOS-type satellites

Since our Newtonian model is based on VMH hypothesis (varying M), now it is worthy to discuss some recent studies to find evidences of expanding Earth, at least in the past few decades. Interestingly, at least there is one study which seem quite worthy to cite here as reported in ANNALS OF GEOPHYSICS, 54, 4, 2011. That paper began with abstract as follows:[46]

“The Earth expansion problem has attracted great interest, and the present study demonstrates that the Earth has been expanding, at least over the recent several decades. Space-geodetic data recorded at stations distributed globally were used (including global positioning system data, very-long baseline interferometry, satellite laser ranging stations, and stations for Doppler orbitography and *radio positioning integrated by satellite*), which covered a period of more than 10 years in the International Terrestrial Reference Frame 2008. A triangular network covering the surface of the Earth was thus constructed based on the spherical Delaunay approach, and average-weighted vertical variations in the Earth surface were estimated. Calculations show that the Earth is expanding at present at a rate of  $0.24 \pm 0.04$  mm/yr. Furthermore, based on the Earth Gravitational Model 2008 and the secular variation rates of the second-degree coefficients estimated by satellite laser ranging and Earth mean-pole data, the principal inertia moments of the Earth (A, B, C) and in particular their temporal variations, were determined: the simple mean value of the three principal inertia moments (i.e.,  $[A+B+C]/3$ ) is gradually increasing. This clearly demonstrates that the Earth has been expanding, at least over the recent decades, and the data show that the Earth is expanding at a rate ranging from  $0.17 \pm 0.02$  mm/yr to  $0.21 \pm 0.02$  mm/yr, which coincides with the space geodetic evidence. Hence, based on both space geodetic observations and gravimetric data, we conclude that the Earth has been expanding at a rate of about 0.2 mm/yr over recent decades.”

A more recent study by the same author concludes: [47]

“According to the space-geodetic data recorded at globally distributed stations over solid land spanning a period of more than 20-years under the International Terrestrial Reference Frame 2008, our previous estimate of the average-weighted vertical variation of the Earth's solid surface suggests that the Earth's solid part is expanding at a rate of  $0.24 \pm 0.05$  mm/a in recent two decades.”

Therefore, it seems that our VMH model of solar system's spiralling gravity is accompanied by Earth's increasing radius too.

In this regard, perhaps one of the most obvious methods to observe those tad effects as predicted by Newtonian dynamics with VMH is using LAGEOS-type satellites, which have already been used to verify Lense-Thirring effect of Earth. What is presented here is merely an approximation, neglecting higher order effects [12][16][31].

Using equation (8c) we could find the rotational effect to satellite orbiting the Earth. Supposed we want to measure the precessional period of the inclined orbit period. Then the best way to measure quadrupole moment ( $J_2$ ) effect would be to measure the  $\vartheta$  component of the gravity force (8c):

$$g = 1/r \cdot \partial V / \partial \vartheta = -3GM \cdot a^2 J_2 \cdot \sin \vartheta \cdot \cos \vartheta / r^4 \quad (25)$$

This component of force will apply a torque to the orbital angular momentum and it should be averaged over the orbit. This yields a known equation, which is often used in satellite observation:

$$\omega_p / \omega_s = -3a^2 J_2 \cdot \cos i / 2r^2 \quad (26)$$

where  $i$  is the inclination of the satellite orbit with respect to the equatorial plane,  $a$  is Earth radius,  $r$  is orbit radius of the satellite,  $\omega_s$  is the orbit frequency of the satellite, and  $\omega_p$  is the precession frequency of the orbit plane in inertial space. Now using LAGEOS satellite data [31] as presented in Table 4:

Table 4. LAGEOS satellite parameters

Parameter	Value	Unit
$R_{\text{LAGEOS}}$	$12.265 \times 10^6$	M
$i_{\text{LAGEOS}}$	109.8	°
$T_{\text{LAGEOS}}$	13673.4	sec
$\omega_s$	$4.595 \times 10^{-4}$	rad/s
$J_2$	$1.08 \times 10^{-3}$	

Inserting this data into equation (26) yields a known value:

$$\omega_p = 0.337561^\circ / \text{day} \quad (27)$$

which is near enough to the observed LAGEOS precession = 0.343°/day.

Now let suppose we want to get an estimate of the effect of Earth kinetic expansion to LAGEOS precession. Assuming a solid sphere, we start with a known equation [34]:

$$M = 4\pi \cdot \rho_{\text{sphere}} \cdot r^3 / 3 \quad (28)$$

where  $\rho_{\text{sphere}}$  is the average density of the 'equivalent' solid sphere. For Earth data (Table 1), we get  $\rho_{\text{sphere}} = 5.50 \times 10^6 \text{ gr/m}^3$ . Using the same method with equation (8f), equation (28) could be rewritten as:

$$M + \Delta M / \Delta t = 4\pi \cdot \rho_{\text{sphere}} \cdot (r + \Delta r / \Delta t)^3 / 3 \quad (29)$$

or

$$\Delta r / \Delta t = \sqrt[3]{(M + \Delta M / \Delta t) \cdot 3 / (4\pi \cdot \rho_{\text{sphere}})} - r \quad (30)$$

From equation (30) we get  $dr/dt = 13.36 \text{ mm/year}$  for Earth. Inserting this value ( $r + dr/dt$ ) to compute back equation (26) yields:

$$\Delta \omega_p = \omega_{p,n+1} - \omega_{p,n} = 1.41 \times 10^{-9} / \text{day} = 2.558 \text{ arc sec} / \text{year} \quad (31)$$

Therefore, provided the aforementioned propositions correspond to the facts, it could be expected to find a tad extra precession of LAGEOS-satellite around 2.558 arcsecond/year. To this author's knowledge this tad effect has not been presented before elsewhere. And also thus far there is no coherent explanation of those aforementioned phenomena altogether, except perhaps in [21] and [30].

As an alternative to this method, it could be expected to observe Earth gravitational acceleration change due to its radius increment. By using equation (28):

$$\ddot{r}(t) = GM / r^2 = 4\pi \cdot G \cdot \rho_{\text{sphere}} \cdot r / 3 \quad (32)$$

From this equation, supposing there is linear radius increment, then we get an expression of the rate of change of the gravitational acceleration:

$$\ddot{r}(t) = \Delta \dot{r} / \Delta t = 4\pi \cdot G \cdot \rho_{\text{sphere}} \cdot (r + \Delta r / \Delta t) / 3 - \ddot{r}(t) \quad (33)$$

It would be interesting to find observation data to verify or refute this equation.

A few more remark shall be made here: although some reports seem to verify the usefulness of GTR view on LAGEOS satellite [48], it seems worthy to check also with other gravity theories, for instance Ronald Hatch's new gravitation model which was developed based on his long experience in GPS technology.[49]

## Discussion

Allow us to make some more remarks: one of the most relevant being the inconstancy of the "constants". Following Dirac's suggestion, the various constants should be validated at period intervals, with no assumption that the values will remain exactly the same as they were at the last measurement.

The fine structure "constant"  $\alpha$ , is the most vulnerable to changes in other physical parameters, because it is relational. Instrumented observations of the *fine structure constant have recorded changes* in the value starting during the 1980s, and with divergences becoming larger than the original value at the time of its inception by Sommerfeld, as time passes.

The fine structure "constant" is a variable,  $G$  is a variable,  $c$  is a variable, and if  $e$  is variable, the normal understandings of the standard physics will be soon be demolished.

Where we live, the laws of physics **are changing** in a gradient manner, along with many of the "constants". This gradient of changing values will continue until the entire of the aether shell has passed through our solar system. At that point, the constants will remain stable in value, and the physics will change in a reliable manner, until the next episode of galactic core-plasma aether ejection occurs and passes through where we live.

The entire process of changing "constants" will start again, and continue until the shell has passed through our location, when the values will once more stabilize and the local physics will again be reliable. Until the next galactic core-plasmoid event.

## Concluding note

If physical theories could be regarded as a *continuing search* to find systematic methods to reduce the entropy required to do calculations to minimum; then the fewer free parameters, the better is the method. This is our slightly modernized interpretation of Occam razor. In this context, the Earth expansion problem has attracted great interest, and recent study gives geodetic evidence that the Earth has been expanding, at least over the recent several decades. In the meantime, Jayant Narlikar & Halton Arp have argued on possible variable mass hypothesis cosmology (VMH). Therefore, in the present article discusses some interesting effects related to varying  $G$ , but here we argue that instead of varying  $G$  we can think of varying mass ( $M$ ).

Among other things we discuss receding planets from the Sun, calculation of Pioneer spacecraft anomaly as proposed by B.G. Sidharth, and also possible slight change in LAGEOS Satellite. Further observation to verify or refute this conjecture is recommended, especially based on geological evidence of expanding earth.

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VC & FS

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