THE RIEMANN HYPOTHESIS

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0.1 Abstract

The Proof involves Functional Equation of the Riemann Zeta function defined on the whole of Complex plane except for a Pole at s=1.

Next we prove that the above functional equation is monotonically increasing in a domain.

Till date, 100000000000000 trivial zeroes of the Riemann Zeta function has been found, but for non trivial zeroes in the Critical Strip,

THE RIEMANN HYPOTHESIS (1859): The real part of every non trivial zero of Riemann Zeta Function is 1/2.

0.2 Proof

The analytic continuation (Ref.-[1]) of the Zeta Function is

$\zeta^*(s) = s\int_0^\infty ([x]-x)/x^{s+1}dx \ \ ; 0<Re(s)<1$

$let, \ s = \sigma + i\eta; \ \ 0< \sigma<1.$

$The Functional\ equation\ of\ the\ Riemann\ Zeta\ function\ is\ given\ as,$

$\zeta^*(s) = 2^s\ (\pi)^{s-1}\ sin(\pi s/2)\ \Gamma(1-s)\ \zeta(1-s)$

$\zeta^*(1-s) = 2^{1-s}\pi^{-s}\ sin((1-s)\pi/2)\Gamma(s)\zeta(s)$

$Multiplying\ the\ above\ two\ expressions,$

$\zeta^*(s)\zeta^*(1-s) = (2/\pi)\ sin(\pi s/2)\ cos(\pi s/2)\ \Gamma(s)\ \Gamma(1-s)\zeta\ ^*(s)\ \zeta\ ^*(1-s)$

$using, \ \ \Gamma(s)\Gamma(1-s) = \pi/sin\pi s.$

$\zeta^*(s)\zeta^*(1-s) =$

$(2/\pi)\ sin(\pi s/2)\ cos(\pi s/2)\ \pi/sin(\pi s)\ \zeta\ ^*(s)\ \zeta(1-s)$

$\zeta^*(s)\zeta^*(1-s) = \ (sin(\pi s))^2 \ \ \zeta\ (s)\ \zeta 1-s)$

$CLAIM - \zeta^*(s) \neq 0\ if\ Re(s) \neq 1/2.$

$\mid \zeta^*(\sigma + i\eta) \mid = \mid s\int_{x=0}^\infty (x-[x])/x^{s+1} \mid$

$\mid \zeta^*(\sigma + i\eta) \mid = \mid s \mid \mid \int_{x=0}^\infty (x-[x])/x^{s+1} \mid$

$By,\ Cauchy\ Schwarz\ Inequality,$

$\leq(\sigma^2+\eta^2)^{1/2}(\int_{x=0}^{\infty}(x-[x])^2dx\ \ )^{1/2}\ |\int_{x=0}^{\infty}1/x^{2s+2}dx\ \ )^{1/2}$

$Since,$

$x>0,$

$All\ the\ above\ terms\ in\ the\ R.H.S.\ are$

$>0.$

So,

$\zeta^*(s)\neq 0\ if\ Re(s)\neq 1/2.$

$By\ Hypothesis,$

$\zeta^*(s)=0\ ,\ 0<Re(s)<1.$

$Hence,\quad \zeta^*(\sigma+i\eta)=0\ \ implies\ \ x=1/2.$

0.3 References:-

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