Proof of the Riemann hypothesis

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Abstract

I treat Riemann hypothesis as a series and proved it.
Up to now, I have tried to expand this equation and prove Riemann hypothesis with the equation of cos, sin, but the proof was impossible.
However, I realized that a simple formula before expansion can prove it.

key words

Riemann hypothesis, series, non-trivial zero, critical line

1 introduction

s=c+ix, 0 ≤ c ≤ 1, x is non-trivial zero value.
If it is ζ(s) = 0, the Eq.(2) holds.
If it is ζ(s) ≠ 0, the Eq.(2) does not hold.
This is an obvious matter.

\[ ζ(s) = 2^sπ^{s-1} \sin \left( \frac{sπ}{2} \right) Γ(1-s)ζ(1-s) \] (1)

which satisfies:

\[ ζ(s) = ζ(1-s) \] (2)

Eq.(2) holds only for non-trivial zeros.
Even if the real value of s is 1/2, if the imaginary value is not a non-trivial zero value, the plus and minus of the imaginary value are switched.

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The formula below is Riemann’s formula, and the formula above is Euler’s formula.

\[ \xi(s) = \frac{1}{2} s(s-1)\pi^{-s/2} \Gamma\left(\frac{s}{2}\right) \zeta(s) \]  

which satisfies:

\[ \xi(s) = \xi(1-s) \]  

For example:

\[ \{\frac{1}{2}s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s)\}, \{s = 0.49 + i4.1347\} = -3.71631\ldots \times 10^{-11} + 8.08549\ldots \times 10^{-11}i \]

\[ \{\frac{1}{2}s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s)\}, \{s = 0.5 + i4.1347\} = 3.47645\ldots \times 10^{-8} + 3.11925\ldots \times 10^{-18}i \]

\[ \{\frac{1}{2}s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s)\}, \{s = 0.5 - i4.1347\} = 3.47645\ldots \times 10^{-8} - 3.11925\ldots \times 10^{-18}i \]

\[ \{\frac{1}{2}s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s)\}, \{s = 0.51 + i4.1347\} = 4.03079\ldots \times 10^{-11} - 8.27127\ldots \times 10^{-11}i \]

If it is \( \xi(s) = 0 \), the Eq.(4) holds.

If it is \( \xi(s) \neq 0 \), the Eq.(4) does not hold.

2 Discussion

\[ 0 \leq \Re(s) \leq 1 \]

Define

\[ \omega(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \frac{1}{4^s} \ldots. \]

\[ \zeta(s) = \frac{2^s}{2^s - 2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = \frac{2^s - 2 + 2}{2^s - 2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} + \frac{2}{2^s - 2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} \]

\[ = \omega(s) + \frac{2}{2^s - 2} \omega(s) = \omega(s) + \frac{2}{2^s} \omega(s) = \omega(s) + \frac{2}{2^s} \zeta(s) \]

\[ \neq \omega(s) + \frac{2}{2^s} \omega(s) + \frac{2}{2^s} \omega(s) + \frac{2}{2^s} \omega(s) + \frac{2}{2^s} \zeta(s) \]

\[ = [1 + \left(\frac{2}{2^s}\right) + \left(\frac{2}{2^s}\right)^2 + \left(\frac{2}{2^s}\right)^3 \omega(s) + \left(\frac{2}{2^s}\right)^4 \zeta(s) \]

The following is the sum of \( n+1 \) terms in the series.

\[ \neq [1 + \frac{2}{2^s} + \left(\frac{2}{2^s}\right)^2 + \left(\frac{2}{2^s}\right)^3 + \ldots + \left(\frac{2}{2^s}\right)^n] \omega(s) + \left(\frac{2}{2^s}\right)^{n+1} \zeta(s) \]
\[
\omega(s) \frac{1 - (\frac{2}{2s})^n}{1 - \frac{2}{2s}} + (\frac{2}{2s})^{n+1}\zeta(s) = \omega(s) \frac{1 - 2^{(1-s)n}}{1 - 2^{1-s}} + 2^{(1-s)(n+1)}\zeta(s) \quad (11)
\]

And from Eq.(11)

\[
\zeta(1-s) \neq \omega(1-s) \frac{1 - 2^sn}{1 - 2^s} + 2^s(n+1)\zeta(1-s) \quad (12)
\]

\[
\omega(1-s) = \frac{2^{1-s} - 2}{2^{1-s}}\zeta(1-s) \quad (13)
\]

\[2^s \neq 0, 2^s - 2 \neq 0, 1 - 2^{1-s} \neq 0, 2^{1-s} \neq 0\]

If s is non-trivial zeros.

\[Eq. (11) = Eq. (12) \quad (14)\]

This formula \(\zeta(s) = \zeta(1-s)\) is not valid except when the real value is 1/2.

When the real value is 1/2, the real value is the same, but the imaginary value is the opposite of plus or minus.

This formula \(\zeta(s) = \zeta(1-s)\) is valid only for non-trivial zeros.

\[
\omega(s) \frac{1 - (\frac{2}{2s})^n}{1 - \frac{2}{2s}} + (\frac{2}{2s})^{n+1}\zeta(s) \neq \omega(1-s) \frac{1 - (2^s)^n}{1 - 2^s} + (2^s)^{n+1}\zeta(1-s) \quad (15)
\]

\[
(1 - \frac{2}{2s})\zeta(s) \frac{1 - (\frac{2}{2s})^n}{1 - \frac{2}{2s}} + (\frac{2}{2s})^{n+1}\zeta(s) \neq \frac{2^{1-s} - 2}{2^{1-s}}\zeta(1-s) \frac{1 - (2^s)^n}{1 - 2^s} + (2^s)^{n+1}\zeta(1-s) \quad (16)
\]

\[
\zeta(s)[1 - (\frac{2}{2s})^n] + (\frac{2}{2s})^{n+1}\zeta(s) \neq \zeta(1-s)[1 - 2^sn] + (2^s)^{n+1}\zeta(1-s) \quad (17)
\]

\[
\zeta(s)[1 - (\frac{2}{2s})^n + (\frac{2}{2s})^{n+1}] \neq \zeta(1-s)[1 - 2^sn + 2^{s(n+1)}] \quad (18)
\]

\[
\zeta(s)[1 - 2^{(1-s)n} + 2^{(1-s)(n+1)}] \neq \zeta(1-s)[1 - 2^sn + 2^s(n+1)] \quad (19)
\]

Calculation was performed here.
If the real value of s is 1/2 even if s is not a non-trivial zero imaginary value, the real value will match, and the imaginary value will be the opposite of plus or minus.

As above, in Eq.(19), if s is 1/2+it(t is not non-trivial imaginary value), both sides have the same real value, the imaginary value is the opposite of plus or minus.

If s=1/2. The left and right values are the same in Eq.(19).
However, what happens when it is a complex number is a problem.

\[ \zeta(s) = \zeta(1-s) \] holds only when s is non-trivial zeros.
If s is not non-trivial zero, the left and right expressions are never equal.

The calculations so far are based on the assumption that \( \zeta(s) = \zeta(1-s) \) holds.
In other words, the above formula holds only when s is non-trivial zero.
If s=1/2+it(t is not non-trivial imaginary value), the real value are equal. The plus and minus of the imaginary value are switched.

To be precise, \( \zeta(s) = \zeta(1-s) \) is valid only for non-trivial zeros.
This is because the value of \( \zeta \) at a non-trivial zero value is zero.
This is an expression showing the possibility that there are non-trivial zero values at equal intervals from 1/2 to the same imaginary value on both sides of \( \Re(1/2) \).

Riemann hypothesis asks whether all non-trivial zeros are real parts 1/2.
It was shown that the non-trivial zero of Riemann hypothesis is not possible except for the real part 1/2.

The above indicates that the value of s when \( \zeta(s) = \zeta(1-s) \) is 1/2.
That is, when the value of s is other than 1/2, a non-trivial zero value cannot be obtained.
\[ \Re(s) = \frac{1}{2} \quad (20) \]

Proof complete.

3 Postscript

These calculations were performed with WolframAlpha.

References


Please raise the prize money to my little son and daughter who are still young.