1. A SURPRISING NOTE ABOUT EUCLID'S PARALLEL AXIOM

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ABSTRACT

In this short note an important consequence of my previous paper[5] is investigated.

We reinvestigate EUCLID'S 5th postulate.

I. INTRODUCTION

There is a very rich literature of attempts to prove EUCLIDS 5^{TH} AXIOM. (See some examples at the end of this paper.)

In this short paper we shall show that the 5^{th} AXIOM is a simple consequence of my previous paper.[5]

II.

One form of Euclid's parallel axiom: through the point P, which is not on line l, one and only one parallel (not intersecting) line can be drawn.

The author of the present short note has proved the following (see[5]): if on the 9. figure of Bolyai's APPENDIX (see at the end of this short note, the figure and the number of the figure is from Bolyai's original APPENDIX) we take a fourth plane which has dihedral right angle with the ABMN plane, then

- the intersection lines of this fourth plane with the AMP and the BNE planes are intersecting each other,
- the BNE plane has a dihedral angle with AMBN less than right angle,
- the dihedral angle at which AMP and AMBN planes intersect is a right angle.
- Then it follows: the fourth plane is an Euclidean plane.

It is easy to see that for every two parallel lines (in Bolyai's sense) we can construct the appropriate planes, lines and points according to the 9. Figure of the APPENDIX in a way that the two parallel lines will be the intersection lines of the new ABNM and the new AMP resp. the new AMNB and the new BNE planes.

In a little more detail: (i) we can always take two parallel lines in Bolyai's sense, mv1 and mv2 (see Figure 2.) and (ii) can always fit to these lines two planes containing (2nd and 3rd planes) these lines. (See 2. Figure). (iii) One of them has right dihedral angle with the 4th plane, and the other has less then right dihedral angle with the 4th plane, and (iv) the intersection lines (mv1 and mv2 on Figure 2.) with the 4th of the 2nd respectively the 3rd planes are the original parallel lines in the 4th plane.

It follows that any parallel pair of lines are parallel in an Euclidean sense also. (See the authors previous paper, see[5]) While the 2nd and 3rd planes are intersecting then mv1 and mv2 lines are also intersecting.

III. A LITTLE HISTORY

There are innumerable attempts to prove Euclid's famous axiom on the basis of the others, perhaps the most famous is Saccheri's quadrangle[4].

Farkas Wolfgang Bolyai also made an unsuccessful attempt to prove the fifth axiom, but Gauss recognized the error, thus W. Bolyai has not published it.[1]

„The father gives a brief resumé of the results of his own determined, life-long, desperate efforts to do that at which Saccheri, J. H. Lambert, Gauss also had failed, to establish Euclid's theory of parallels a priori. He says, p. 490: "tentamina idcirco quae olim feceram, breviter expo-nenda veniunt; ne saltem alius quis operam eandem perdat."' He anticipates J. Delboeufs "Prolégoménes philosophiques de la géométrie et solution des postu-lato," with the full consciousness in addition that it is not the solution,-that the final solution has crowned not his own intense efforts, but the genius of his son."[6???]

Two further attempts of proving Euclid's famous parallel postulate have been made by B. F. Thibaut in 1809, to be found in Marvin Jay Greenberg's book [3] as exercise 52 of chapter 9, as well as a letter from the Danish mathematician Schumacher to Carl Friedrich Gauss originally written in 1831, and published in Vol. 8. of The Collected Works of Gauss [2].

IV. REFERENCES

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Does another Euclidean plane exist other than the parasphere?

6. Wikipedia, https://en.wikipedia.org/wiki/Farkas_Bolyai

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§9

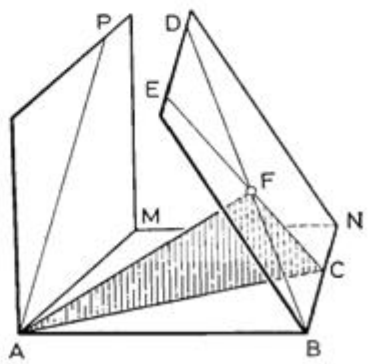

Figure 9

COMMENTS TO THE 2. FIGURE

The A’M’P’ plane has fit to the mv1 line.

The B’N’E’ plane has fit to the mv2 line.

The dihedral angle of the 2^{nd} plane has to be right angle with the 4^{th} plane, while that of the 3^{rd} plane has to be less then right angle with the 4^{th} plane.

The 4^{th} plane has right dihedral angle with the A’M’B’N’ plane and has to contain the mv1 and mv2 lines.

As the A'M'P' plane and the B'N'E' plane according to the APPENDIX 9. paragraph intersect while the dihedral angle of the A'M'B'N' and B'N'E' planes less then right angle (and the A'M'B'N' and A'M'P' planes have right angle) it is clear that the intersection line mv1 of the 2nd and the 4th planes and the intersection line mv2 of the 3rd and the 4th planes are also intersecting. Consequently the 4th plane is Euclidean, and mv1, mv2 were arbitrary, in Bolyai's sense parallel lines!

(The 2nd plane is the A'M'P' plane while the 3rd plane is the B'N'E' plane.)

FIGURE 2