Causality Between Events with Space-Like Separation

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There was a young lady named Bright
Whose speed was much faster than light.
She left one day, in a relative way,
And returned the previous night.
-attributed to A. H. R. Buller, 1923

Since the first part of the twentieth century, it has been maintained that faster-than-light movement could produce time travel into the past with its accompanying causality-violation paradoxes. This paper demonstrates that this assumption is false because of misinterpretation of the Minkowski diagram and the Lorentz transformation (LT) upon which it is based, plus the completely unsubstantiated belief that the past is "back there somewhere."

1.0 Introduction

G. Feinberg coined the name "tachyon"[1] for a particle that always travels faster than light, satisfies the principle of relativity and is Lorentz-invariant. The limiting value is c, but, as Feinberg points out, a limit has two sides. There has been some evidence that neutrinos are tachyons, recent data being $m^2 = -0.6\ eV^2/c^4$ with substantial error bars.[2] Not a very strong recommendation for tachyonic neutrinos, but there are other possibilities for getting from point A to point B faster than light can do it. The purpose of this paper is to investigate whether or not such processes would violate known physics or causality.

In 1907 A. Einstein considered it to be "sufficiently proven" that any velocity greater than that of light is an impossibility[3] by analysis of relativistic velocity composition and the Lorentz transformation equation for time. Given an inertial frame moving at velocity v with respect to a "stationary" frame, the time differential over a distance L is

$$\Delta t' = \gamma(\Delta t - vL/c^2) \qquad (1)$$

He concluded that for Δt less than vL/c^2, $\Delta t'$ would be negative, implying that any such speedy object would arrive at its destination before it departed from its origination point. Similarly, R. C. Tolman pointed out in 1917 that velocities greater than the speed of light presented the possibility that effect could precede cause.[4]

The assertion that causality can be violated by faster-than-light travel is also mainstream thought in this century. N. D. Mermin[5] wrote, "In the [moving] frame …the object is in two different places at the same time! This is such a bizarre situation that one's suspicion is strengthened that the difficulty we have already encountered in producing an object moving faster than light must be a reflection of the impossibility of such motion." This is another aspect of a causality violation, but perhaps the "impossibility" is not in the movement of such an object but, rather, in misinterpreting the LT in such a way that it does not agree with the reality of our world which is governed by entropy and the "arrow of time."

The purpose of this paper is to demonstrate that the minus sign in Equation (1) sets a limit on speeds observed in relatively-moving inertial frames. When $\Delta t = vL/c^2e$, $\Delta t' = 0$, thus the velocity of an object so described will be $u' = \Delta x'/\Delta t' = \infty$ in one frame but c^2/v in a different frame, where v is the velocity difference between the two frames. This prevents the bizarre absurdities of going backward in time and bringing multiple objects into existence which are purported to occur with superluminal movement.

2.0 The Minkowski Diagram.

The Minkowski diagram is a simple time-position representation of a stationary frame with a moving frame, determined from the Lorentz transformation equations, superimposed upon it. Consequently, all time and position values are viewed from the stationary frame, not the moving frame. "Moving" and "stationary" are completely arbitrary, but we will call the "stationary" frame the one in which A and B are stationary and the "moving" frame the one in which C and D are stationary.

Figure 1 shows a typical Minkowski diagram, a graphical representation of the Lorentz transform. The x = 0 and x = L vertical lines represent the trajectories, or "world lines" of A and B, stationary objects in the stationary frame (shown in blue). Objects C and D are moving at some velocity, v, with respect to A and B, where v is less than c. The axes of the moving frame, x' and t', are tilted with respect to the stationary frame, the x' axis of the moving frame being defined by $t = vx/c^2$, where t and x are coordinates of the stationary frame.

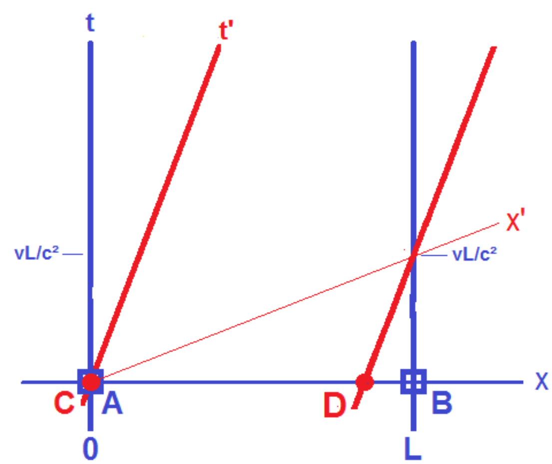

Figure 1. Typical Minkowski Diagram

The trajectories of C and D in the moving frame are defined by x= vt and $x = L(1 - v^2/c^2) + vt$, respectively, where x and t are, once again, coordinates of the stationary frame. The positions of A, B, C and D at t = 0 are shown in Figure 1. All objects advance along their trajectories as t advances, but according to the mainstream view, A and C are still back at t = 0, as depicted in Figure 2. This assumes that the past is somehow real and accessible. According to this view, B originates a signal and transfers it to D at $t = vL/c^2$ when they are adjacent at Event E1 and D transfers the signal to C instantaneously in their moving frame, as shown by the downward-sloping black arrow.

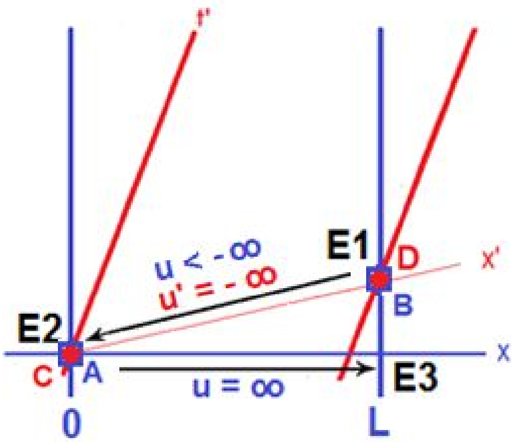

Figure 2. Typical Minkowski Diagram with objects A and C at t = 0 and B and D at $t = vL/c^2$, Showing Purported Causality Violation

Since $t_D' = 0$ when $t = vL/c^2$, it should arrive when $t_C' = 0$, which is when t = 0 and x = 0 at Event E2. Thus when A sends the signal back to B instantaneously, it arrives there at t = 0 (Event E3), before B sends it at Event E1. This means that B at t = vL/c2 could not have originated the signal in the first place, hence, a causality violation.

The arrow in Figure 2 labeled "u < -∞" is not an error. In Minkowski diagrams velocity is represented by an angle. Figure 3 depicts angles for speeds varying from small to large as the angle increases counterclockwise. Infinite speed is represented by a horizontal line (a distance displacement in zero time) and it is absurd to argue that a speed can be even greater than infinity, yet that is exactly what the mainstream view does.

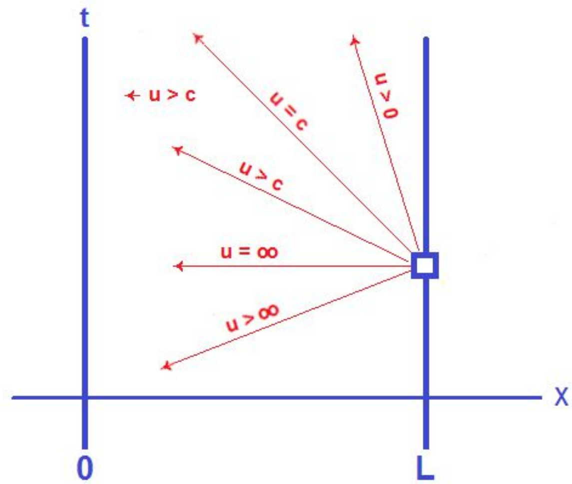

Figure 3. Minkowski Diagram Showing Directions of Various Speeds

According to the correct Minkowski diagram shown in Figure 4, C is no longer at t = 0 since A and C should have also advanced to t = vL/c2. All experimental evidence says that only the present exists: no one has sent a signal or other object into the past. All claims to the contrary are in the domain of science fiction fantasies. Consequently, D cannot send a signal back to x = 0, t = 0 in the stationary frame at a speed faster than infinity in *any* frame.

3.0 The Case for Causality

In the mainstream view of Figure 2, advancement in time of A and C has been suppressed while B and D have been advanced, That model makes the unwarranted assumption that the past actually exists whereas there is absolutely no evidence that it does. In fact, all evidence points to the past only existing in memory of one kind or another (rocks, tree rings, silicon, neurons, etc.). This agrees with the philosophy of the ontology of time called “presentism”[6] and is adamantly opposed to the “block universe” concept.[7]

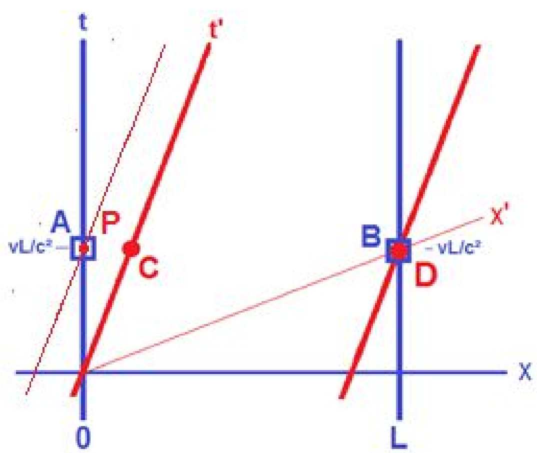

Figure 4. Correct Minkowski Diagram of Objects In Figure 1 at $t = vL/c^2$

As Figure 4 shows, A and C are no longer at t = 0 when B and D are at $t = vL/c^2$. It’s not superluminal motion that leads to the conclusions Mermin found so bizarre, it’s the claim that A and C are still at t = 0. No, when D is at $t = vL/c^2$, C is also, as shown in Figure 4.

Another small problem with Figure 4 is also apparent: at $t = vL/c^2$, A and C are no longer adjacent. This is minor, however, since another point can be presumed, call it P, in the moving frame to the left of C such that it will intersect A at point $t = vL/c^2$ (as shown in Figure 4), or some point greater, as will be demonstrated in the next section.

4.0 Relativistic Velocity Composition

When a frame moving at velocity v with respect to a stationary frame sends out a signal or object at velocity u’ (with respect to the moving frame), the velocity of said signal or object with respect to the stationary frame is[8]

$$u = (u' + v)/(1 + u'v/c^2) \qquad (2)$$

This equation demonstrates the invariance of c, the speed of light. If either u’ or v are equal to c, u will also be equal to c. What is not always recognized, however, is that while the derivation of Equation (2) doesn’t in any way limit velocities to those less than or equal to c, it also applies to hypotheses about speeds faster than that of light, and that what is infinite velocity is one frame is not infinite in a different frame. As Feinberg said, a limit has two sides.[1] Suppose, for example, we let u’ go to infinity. Then

$$u = c^2/v \qquad (3)$$

This has a significant impact on purported demonstrations of causality violation in the literature.[3,4,5,9,10] These scenarios involve signals exceeding the speed of light in both directions along the spatial axes, and they assume that the value of any faster-than-light speed is preserved in other frames. Equations (2) and (3) refute this assumption. These equations are derived from u’ and v both being in the positive direction. Because space is homogeneous and isotropic, we know that when u’ and v are both in the negative direction, Equation (3) is correct when v is replaced by –v.

Besides the scenario where both u' and v are in the same direction, designated Case I, there is Case II where u' and v are in opposite directions; however, it should be pointed out that the inverse of the LT is obtained by simply reversing the sign of v. This operation makes the "moving" frame the "stationary" frame and the "stationary" frame the "moving frame." Consequently, Case II is merely the same as Case I but from a different perspective.

In order to correctly analyze the example depicted in Figure 4, which is Case II, it is necessary to examine the problem with the Lorentz transformation. The parameters are

$$x_A = 0 \qquad t_A = L/(-u) + vL/c^2$$
$$x_B = L \qquad t_B = vL/c^2$$
$$x_E' = \gamma(0 - vt_A)$$
$$t_E' = \gamma[L/(-u) + vL/c^2]$$
$$x_D' = \gamma(L - vt_B)$$
$$t_D' = \gamma(t_B - vL/c^2) = 0$$

where the moving frame moves to the right and u is the velocity of an object as observed in the "stationary" frame moving to the left (represented by a negative number). The velocity of the object in the moving frame is

$$u' = \Delta t'/\Delta x'$$

$$u' = (x_E' - x_D')/(t_E' - t_D')$$

$$u' = \gamma[- vL/(-u) - v^2L/c^2 - L + v^2L/c^2]/\gamma[L/(-u) + vL/c^2]$$

$$u' = [- vL/(-u) - L]/[L/(-u) + vL/c^2]$$

$$u' = - [(-u) + v]/[1 + (-u)v/c^2] \qquad (4)$$

where (-u) will be a positive number. Letting (-u) grow without limit,

$$u' = - c^2/v \qquad (5)$$

If the magnitude of u' were to grow without limit, the superluminal object would indeed arrive at $t' = x' = 0$, but it takes time $\gamma L/(-u') = \gamma vL/c^2$, which is just vL/c^2 in the stationary frame, as shown in Figure 5. The signal transferred from B to D at $t = vL/c^2$ and D transfers it to P in the moving frame, arriving when P is at $t = vL/c^2$ in the stationary frame. The signal is transferred to A, which is adjacent to P, and then from A to B instantaneously. Thus the signal arrives back at B the moment it was originally sent and there is *no* backward-in-time anomaly and *no* causality violation.

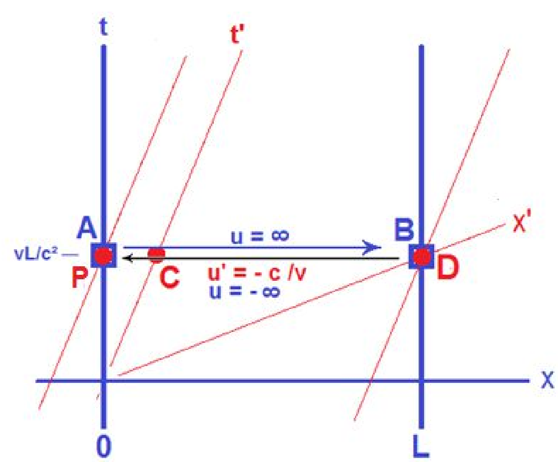

Figure 5. Correct Minkowski Diagram Showing Causality Violation Cannot Occur

Figure 5 describes the situation when the signal is instantaneous from the perspective of the stationary frame. It's speed is $-c^2/v$ with respect to the moving frame. To find how fast the signal moves in the stationary frame when it is instantaneous in the moving frame, we use the inverse Lorentz transform. This is accomplished by replacing v with –v, which makes both v and u' in the same direction, which is Case I, and we already know that the answer is:

$$u = -c^2/v \tag{5}$$

This means that the signal will arrive at P and be transferred to A at t = 2vL/c2 and get back to B even later, which still has no causality violation, of course.

5.0 Conclusion

Relativistic velocity composition is a consequence of the Lorentz transformation equations and demonstrates the fallacy that superluminal signals allow causality violations. The c^2/v limit prevents backward-in-time phenomena for all moving frame velocities, causing the observed superluminal velocity to be lower and lower as the moving frames move faster and faster, approaching c from the upper side as v approaches c from the lower side. Consequently, there is no theoretical objection to superluminal communication, although there is no solid experimental evidence for such.

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6.0 References

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