A BOUND FOR THE ISOTROPIC CONSTANT IN THE
SYMMETRIC CASE

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Abstract. In this preprint we will prove an explicit bound for the
isotropic constant in the symmetric case.

1. Introduction

We say that a convex body $K$ is centralized, if

$$0 = \int_K \langle x, \phi \rangle,$$

for all $\phi \in S^{n-1}$. The entries of the covariance matrix of a convex body $K$
are defined as

$$(a_{ij}) = \frac{\int_K x_i x_j}{|K|} - \frac{\int_K x_i}{|K|} \frac{\int_K x_j}{|K|}.$$

We define the isotropic constant of any convex body $K$ via

$$I^n_K := \frac{\text{Det}(\text{Cov}K)}{|K|^2}.$$

[2]. We define the polar of $K$ as

$$K^\circ := \{x \in \mathbb{R}^n | \langle x, y \rangle \leq 1 \text{ for all } y \in K\}.$$ 

The Mahler volume $s(K)$ of $K$ is defined as

$$s(K) := |K| * |K^\circ|.$$ 

We say that the convex body is in isotropic position if it is centralized and the
 covariance matrix is a constant times the unit matrix. This kind of position
exists [4]. The reverse Santaló inequality says that there is an universal
constant $c$ such that

$$c^n |B_n|^2 \leq |K| |K^\circ|,$$

where $B_n$ is the $n$-dimensional euclidean unit ball [1]. The isotropic constant
for a ball $L_{B_n}$ is well know to be bounded. We will prove

Theorem 1. For all isotropic symmetric convex bodies $K$ it holds that

$$L_K \leq L_{B_n}^{-1} (s(K))^{-1/n} \leq C,$$

where $C$ is an universal constant.

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2. The proof of the main theorem

Let $K$ be an unit ball. Then there exist $T(K), T \in SLG(n, \mathbb{R}^n)$, that is isotropic and the covariance matrix is a constant times the unit matrix [4]. Using that $K \subset \sqrt{n}B_2$ we can calculate

\[ L^2_{T(K)} = \frac{\int_{T(K)} |x|^2}{n|K|} * |K|^{-2/n} \leq \frac{|K|^n}{|K|} * |K|^{-2/n} = |K|^{-2/n} \]

and

\[ L^2_{A(K)} = \frac{\int_{A(K)} |x|^2}{n|K|^\circ} * |K|^\circ \leq \frac{|K|^\circ n}{|K|^\circ} * |K|^\circ \leq |K|^\circ \leq 1 \]

where $A \in SLG(n, \mathbb{R}^n)$ and $A(K)$ is isotropic. Thus form (2) and (3) we obtain

\[ L_{B_2} L_{T(K)} \leq L_{T(K)} L_{A(K)} \leq \frac{1}{s(K)\frac{1}{n}}, \]

where we use the fact that $L_{B_2} \leq L_K$ for all convex bodies $K$. Combining (4) with Milman-Bourgain (or reverse Santaló) inequality (1), we obtain

\[ L_{T(K)} \leq \frac{1}{L(B_2)s(K)\frac{1}{n}} \leq C, \]

which implies the main theorem 1.

References


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