.

A BOUND FOR THE ISOTROPIC CONSTANT IN THE SYMMETRIC CASE

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ABSTRACT. In this preprint we will prove an explicit bound for the isotropic constant in the symmetric case.

1. Introduction

We say that a convex body K is centralized, if

$$0 = \int_K \langle x, \phi \rangle,$$

for all $\phi \in S^{n-1}$. The entries of the covariance matrix of a convex body K are defined as

$$(a_{ij}) = \frac{\int_K x_i x_j}{|K|} - \frac{\int_K x_i}{|K|}\frac{\int_K x_j}{|K|}.$$

We define the isotropic constant of any convex body K via

$$L_K^{2n} := \frac{\mathrm{Det}(CovK)}{|K|^2}$$

[2]. We define the polar of K as

$$K^\circ := \{x \in \mathbb{R}^n | \langle x, y \rangle \leq 1 \quad \text{for all} \quad y \in K\}.$$

The Mahler volume $s(K)$ of K is defined as

$$s(K) := |K| * |K^\circ|.$$

We say that the convex body is in isotropic position if it is centralized and the covariance matrix is a constant times the unit matrix. This kind of position exists [4]. The reverse Santaló inequality says that there is an universal constant c such that

$$c^n |B_n|^2 \leq |K||K^\circ|, \tag{1}$$

where B_n is the n-dimensional euclidean unit ball [1]. The isotropic constant for a ball L_{B_n} is well know to be bounded. We will prove

Theorem 1. *For all isotropic symmetric convex bodies K it holds that*

$$L_K \leq L_{B_2}^{-1}(s(K))^{-1/n} \leq C,$$

where C is an universal constant.

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2. The proof of the main theorem

Let K be an unit ball. Then there exist $T(K), T \in SLG(n, \mathbb{R}^n)$, that is isotropic and the covariance matrix is a constant times the unit matrix [4]. Using that $K \subset \sqrt{n}B_2$ we can calculate

$$L_{T(K)}^2 = \frac{\int_{T(K)} |x|^2}{n|K|} * |K|^{-2/n} \leq \frac{|K|n}{|K|n} * |K|^{-2/n} = |K|^{-2/n} \tag{2}$$

and

$$L_{A(K)}^2 = \frac{\int_{A(K)} |x|^2}{n|K|^\circ * |K^\circ|}^{-2/n} \leq \frac{|K^\circ|n}{|K^\circ|n} * |K^\circ|^{-2/n} = |K^\circ|^{-2/n}, \tag{3}$$

where $A \in SLG(n, \mathbb{R}^n)$ and $A(K)$ is isotropic. Thus form (2) and (3) we obtain

$$L_{B_2} L_{T(K)} \leq L_{T(K)} L_{A(K)} \leq \frac{1}{s(K)^{1/n}}, \tag{4}$$

where we use the fact that $L_{B_2} \leq L_K$ for all convex bodies K. Combining (4) with Milman-Bourgain (or reverse Santaló) inequality (1), we obtain

$$L_{T(K)} \leq \frac{1}{L(B_2)s(K)^{1/n}} \leq C,$$

which implies the main theorem 1.

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