

Refutation of the paradox of Hempel's raven

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Abstract: We evaluate the hypothesis which is *not* tautologous and hence *not* a paradox. It forms a *non* tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET \sim Not, \neg ; + Or, \vee , \cup , \sqcup ; - Not Or; & And, \wedge , \cap , \sqcap , \cdot , \otimes ; \ Not And;
 $>$ Imply, greater than, \rightarrow , \Rightarrow , \mapsto , $>$, \supset , \rightarrow ; $<$ Not Imply, less than, \in , $<$, \subset , \prec , \neq , \ll , \lesssim ;
 $=$ Equivalent, \equiv , $:=$, \Leftrightarrow , \leftrightarrow , $\hat{=}$, \approx , \simeq ; @ Not Equivalent, \neq , \oplus ;
 $\%$ possibility, for one or some, \exists , \diamond , M ; # necessity, for every or all, \forall , \square , L ;
 $(z=z)$ **T** as tautology, \top , ordinal 3; $(z@z)$ **F** as contradiction, \emptyset , Null, \perp , zero;
 $(\%z\>\#z)$ **N** as non-contingency, Δ , ordinal 1; $(\%z\<\#z)$ **C** as contingency, ∇ , ordinal 2;
 $\sim(y < x)$ ($x \leq y$), ($x \subseteq y$), ($x \sqsubseteq y$); $(A=B)$ $(A\sim B)$.
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Raven_paradox

Hempel describes the paradox in terms of the hypothesis: ..

(1) All ravens are black. (1.1.1)

In the form of an implication, this can be expressed as:

If something is a raven, then it is black. (1.2.1)

Remark 1.2.1: We write Eq. 1.2.1 as:

If raven, then black. (1.3.1)

LET $p, q, r, s:$ black, green apple, raven, s.
 $r \supset p;$ TTTT **FTFT** TTTT **FTFT** (1.3.2)

Via contraposition, this statement is equivalent to:

(2) If something is not black, then it is not a raven. (2.1.1)

Remark 2.1.1: To map via contraposition, we write Eq. 2.1.1 as:

If not black, then not raven. (2.2.1)

$\sim p \supset \sim r;$ **TFTF** TTTT **TFTF** TTTT (2.2.2)

In all circumstances where (2) is true, (1) is also true—and likewise, in all circumstances where (2) is false (i.e., if a world is imagined in which something that was not black, yet was a raven, existed), (1)

is also false. (2.3.1)

... Given a general statement such as *all ravens are black*, a form of the same statement that refers to a specific observable instance of the general class would typically be considered to constitute evidence for that general statement. For example,

(3) *My pet raven is black.* (3.1.1)

is evidence supporting the hypothesis that *all ravens are black*.

Remark 3.1.1: Eqs. 1.3.1 and 3.1.1 are equivalent.

The paradox arises when this same process is applied to statement (2). On sighting a green apple, one can observe:

(4) *This green apple is not black, and it is not a raven.* (4.1.1)

$(q \supset \sim p) \& (q \supset \sim r)$; TTTT **TTF** TTTT **TTF** (4.1.2)

By the same reasoning, this statement is evidence that (2) *if something is not black then it is not a raven.* (5.1.1)

$((q \supset \sim p) \& (q \supset \sim r)) \supset (\sim p \supset \sim r)$; TTTT **FTTT** TTTT **FTTT** (5.1.2)

Remark 5.1.2: Eq. 5.1.2 is *not* tautologous to mean 4.1.2 is *not* evidence of 2.2.2.

But since (as above) this statement is logically equivalent to (1) *all ravens are black*, it follows that the sight of a green apple is evidence supporting the notion that all ravens are black.

$((((q \supset \sim p) \& (q \supset \sim r)) \supset (\sim p \supset \sim r)) \supset (r \supset p))$; TTTT **TFT** TTTT **TFT** (6.1.1)

This conclusion seems paradoxical because it implies that information has been gained about ravens by looking at an apple.

Remark 6.1.1: Eq. 6.1.1 is *not* tautologous, and it does *not* imply that information was gained about ravens by looking at an apple. Hence the hypothesis is *not* a paradox.