

## Refutation of the paradox of Hempel's raven

© Copyright 2019 by Colin James III All rights reserved.

**Abstract:** We evaluate the hypothesis which is *not* tautologous and hence *not* a paradox. It forms a *non* tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET  $\sim$  Not,  $\neg$ ; + Or,  $\vee$ ,  $\cup$ ,  $\sqcup$ ; - Not Or; & And,  $\wedge$ ,  $\cap$ ,  $\sqcap$ ,  $\cdot$ ,  $\otimes$ ; \ Not And;  
 $>$  Imply, greater than,  $\rightarrow$ ,  $\Rightarrow$ ,  $\mapsto$ ,  $>$ ,  $\supset$ ,  $\rightarrow$ ;  $<$  Not Imply, less than,  $\in$ ,  $<$ ,  $\subset$ ,  $\prec$ ,  $\neq$ ,  $\ll$ ,  $\leq$ ;  
 $=$  Equivalent,  $\equiv$ ,  $:=$ ,  $\Leftrightarrow$ ,  $\leftrightarrow$ ,  $\hat{=}$ ,  $\approx$ ,  $\simeq$ ; @ Not Equivalent,  $\neq$ ,  $\oplus$ ;  
 $\%$  possibility, for one or some,  $\exists$ ,  $\diamond$ ,  $M$ ; # necessity, for every or all,  $\forall$ ,  $\square$ ,  $L$ ;  
 $(z=z)$  **T** as tautology,  $\top$ , ordinal 3;  $(z@z)$  **F** as contradiction,  $\emptyset$ , Null,  $\perp$ , zero;  
 $(\%z\>\#z)$  **N** as non-contingency,  $\Delta$ , ordinal 1;  $(\%z\<\#z)$  **C** as contingency,  $\nabla$ , ordinal 2;  
 $\sim(y < x)$  ( $x \leq y$ ), ( $x \subseteq y$ ), ( $x \sqsubseteq y$ );  $(A=B)$   $(A\sim B)$ .  
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: [en.wikipedia.org/wiki/Raven\\_paradox](http://en.wikipedia.org/wiki/Raven_paradox)

Hempel describes the paradox in terms of the hypothesis: ..

(1) All ravens are black. (1.1.1)

In the form of an implication, this can be expressed as:

If something is a raven, then it is black. (1.2.1)

**Remark 1.2.1:** We write Eq. 1.2.1 as:

If raven, then black. (1.3.1)

LET  $p, q, r, s:$  black, green apple, raven, s.  
 $r \supset p$ ; TTTT **FTFT** TTTT **FTFT** (1.3.2)

Via contraposition, this statement is equivalent to:

(2) If something is not black, then it is not a raven. (2.1.1)

**Remark 2.1.1:** To map via contraposition, we write Eq. 2.1.1 as:

If not black, then not raven. (2.2.1)

$\sim p \supset \sim r$ ; **TFTF** TTTT **TFTF** TTTT (2.2.2)

In all circumstances where (2) is true, (1) is also true—and likewise, in all circumstances where (2) is false (i.e., if a world is imagined in which something that was not black, yet was a raven, existed), (1)

is also false. (2.3.1)

**Remark 3.1.1:** We write Eq. 2.3.1 in two parts as:

If Eqs. 2.2.2 is true, then 1.3.2 is true; (2.4.1)

$$(\sim p \supset \sim r) \supset (r \supset p); \quad \text{TTTT TTTT TTTT TTTT} \quad (2.4.2)$$

and if 2.2.2 is false, then 1.3.2 is false. (2.5.1)

$$\sim(\sim p \supset \sim r) \supset \sim(r \supset p); \quad \text{TTTT TTTT TTTT TTTT} \quad (2.5.2)$$

$$(((\sim p \supset \sim r) = (s = s)) \supset ((\sim p \supset \sim r) = (s = s))) \& (((\sim p \supset \sim r) = (s @ s)) \supset ((\sim p \supset \sim r) = (s @ s))); \quad \text{TTTT TTTT TTTT TTTT} \quad (2.4.2)$$

Given a general statement such as *all ravens are black*, a form of the same statement that refers to a specific observable instance of the general class would typically be considered to constitute evidence for that general statement. For example,

$$(3) \text{ My pet raven is black.} \quad (3.1.1)$$

is evidence supporting the hypothesis that *all ravens are black*.

**Remark 3.1.1:** Eqs. 1.3.1 and 3.1.1 are equivalent.

The paradox arises when this same process is applied to statement (2). On sighting a green apple, one can observe:

$$(4) \text{ This green apple is not black, and it is not a raven.} \quad (4.1.1)$$

$$(q \supset \sim p) \& (q \supset \sim r); \quad \text{TTTF TTFF TTTF TTFF} \quad (4.1.2)$$

By the same reasoning, this statement is evidence that (2) *if something is not black then it is not a raven*. (5.1.1)

$$((q \supset \sim p) \& (q \supset \sim r)) \supset (\sim p \supset \sim r); \quad \text{TTTT FTTF TTTF FTTF} \quad (5.1.2)$$

**Remark 5.1.2:** Eq. 5.1.2 is *not* tautologous to mean 4.1.2 is *not* evidence of 2.2.2.

But since (as above) this statement is logically equivalent to (1) *all ravens are black*, it follows that the sight of a green apple is evidence supporting the notion that all ravens are black.

$$(((q \supset \sim p) \& (q \supset \sim r)) \supset (\sim p \supset \sim r)) \supset (r \supset p); \quad \text{TTTT TTFT TTTF TTFT} \quad (6.1.1)$$

This conclusion seems paradoxical because it implies that information has been gained about ravens by looking at an apple.

**Remark 6.1.1:** Eq. 6.1.1 is *not* tautologous, and it does *not* imply that information was gained about ravens by looking at an apple. Hence the hypothesis is *not* a paradox.