Refutation of perfect and strong functions in fuzzy logic

Abstract: For fuzzy logic, we evaluate the definition of the residuated lattice operator as *not* tautologous. We evaluate the perfect fuzzy function and strong (surjective) fuzzy function as *not* tautologous, but logical equivalents. This refutes the fuzzy functions and distinctions. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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Abstract The aim of this investigation is to reconsider two notions of fuzzy function, namely: a fuzzy function as a special fuzzy relation and a fuzzy function as a mapping between fuzzy spaces. We propose to combine both notions in such a way that a fuzzy function as a relation determines a fuzzy function as a mapping. We investigate conditions which guarantee that dependent values of the related fuzzy functions coincide. Moreover, we investigate properties and relationship of the related fuzzy functions in the case when both of them are "fuzzified" versions of the same ordinary function.

1. Introduction The notion of fuzzy function has at least two different meanings in fuzzy literature. From theoretical point of view (see e.g., ..), a fuzzy function is a special fuzzy relation with a generalized property of uniqueness. According to this approach, each element from a range of a fuzzy function can be assigned with a certain degree to each element from its domain. Thus, instead of working with direct functional values we have to work with degrees. Another, practical point of view on a fuzzy function originates from the early work of L. Zadeh .. where he proposed the well known extension principle. According to this principle, every function (in an ordinary sense) can be "fuzzified", i.e. extended to arguments given by fuzzy sets. Thus, any ordinary function determines a mapping from a set of fuzzy subsets of its domain to a set of fuzzy subsets of its range. In .. , we have used this approach and defined a fuzzy function as an ordinary mapping between two universes of fuzzy sets. Similar definition appeared in .. and implicitly, in many other papers devoted to fuzzy IF-THEN rules models where these models are used as partially given fuzzy functions. The aim of this investigation is to reconsider both notions and combine them in such a way that a fuzzy function as a relation determines a fuzzy function as a mapping (we will say that they are related). We will investigate conditions which guarantee that dependent values of related fuzzy functions coincide. Moreover, we will investigate properties and relationship of related fuzzy functions in the case when they are "fuzzified" versions of the same ordinary function.

2.2. Residuated lattice ...

Definition 1 A residuated lattice is an algebra … such that … the operation → is a residuation with respect to ∗, i.e. a ∗ b ≤ c iff a ≤ b → c. (2.2.1.1)

LET p, q, r: a, b, c.

(~(q<p)>r)>~(r<(p&q)) ; TTTT **FFF**T TTTT **FFF**T (2.2.1.2)

Remark 2: Eq. 2.2.1.2 is *not* tautologous. Hence the definition for the operation → as a residuation is refuted.

3.1. (E−F)-fuzzy function

...

Definition 3 Let E, F be respective fuzzy equivalences on X and Y . An (E−F)-fuzzy function is a binary fuzzy relation ρ on X × Y such that ... the following axioms hold true: … An (E−F)-fuzzy function is called perfect ... **F.4** for all x∈X, there exists y∈Y, such that ρ(x,y) = 1. (3.1.3.4.1)

LET r, x, y, u, v: ρ, x, y, X, Y.

((#x<u)&(%y<v))>((r&(#x&%y))=(%z>#z)) ;

TTTT TTTT TTTT TTTT } x48
TTTT CCCC TTTT CCCC } x 2 *} x 2*
TTTT TTTT TTTT TTTT } x 6 *}* (3.1.3.4.2)

An (E−F)-fuzzy function is called (strong) surjective ... if **F.5** for all y∈Y, there exists x∈X, such that ρ(x, y) = 1. (3.1.3.5.1)

((#y<v)&(%x<u))>((r&(%x&#y))=(%z>#z)) ;

TTTT TTTT TTTT TTTT } x48
TTTT CCCC TTTT CCCC } x 2 *} x 2*
TTTT TTTT TTTT TTTT } x 6 *}* (3.1.3.5.2)

Remark 3: Eqs. 3.3.4.2 and 3.1.3.5.2 as rendered are *not* tautologous, but produce sames truth table values. Hence the axioms for the (E−Z)-functions as perfect or strong (surjective) are refuted and equivalents.