Refutation of shared variables in cross axiom models of alternating Turing machines

Abstract: We evaluate shared variables for the reduction of alternating Turing machines (ATMs) to subset space logic (SSL). The purpose of shared variables is to use binary counters for mapping cross-axiom models. None is tautologous, to refute cross-axiom models in the completeness conjectures. These form *non* tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Hertling, P.; Krommes, G. (2019). EXPSPACE-completeness of the logics K4×S5 and S4×S5 and the logic of subset spaces, part 2: EXPSPACE-hardness. arxiv.org/pdf/1908.03509.pdf

1 Introduction In this article we are concerned with the complexity of the bimodal product logics K4×S5 and S4×S5 and with the subset space logic SSL, a bimodal logic as well. To the best of our knowledge, the complexity of K4×S5, of S4×S5, and of SSL were open problems.

3 Preparations for the reduction of alternating Turing machines to SSL

3.1 Shared variables We have to make sure that various kinds of information are stored in a suitable way in any model of the fo[r]mula.

Definition 3.1 (Shared Variables). For i ∈ N let A_i be special propositional variables, and let B be another special propositional variable B, different from all A_i . The shared variables α_i are defined as follows:

$\alpha_i := L(A_i \wedge LB)$. Note that $\neg\alpha_i \equiv K(\neg A_i \vee \Diamond K \neg B)$. (3.1.1.1)

LET p, q, r, s: A_i, B, K, L.

~(s&(p&(#s&q)))=(r&(~p&(%r&~q))) ;
FFFF T**FFF** **FFF**N T**FF**N (3.1.1.2)

A.1 Binary Counters in S4×S5 The shared variables α_i are defined as

$\alpha_i := LA_i$. Note that $\neg\alpha_i \equiv K\neg A_i$. (A.1.2.1)

~(s&p)=(q&~p) ; **FF**T**F** **FF**T**F** **F**TTT **F**TTT (A.1.2.2)

Remark 4.0: Eqs. 3.1.1.2 and A.1.2.2 as *not* tautologous or equivalent are both taken as defining shared variables. We write the combined definition of shared variables as 3.1.1.1 And A.1.2.1: (4.1)

(~(s&(p&(#s&q)))=(r&(~p&(%r&~q))))&(~(s&p)=(q&~p)) ;

FFFF FFFF FFFN **FFF**N (4.2)

Remark 4.2: The purpose of shared variables is to use binary counters for mapping cross-axiom models. Because Eq. 4.2 is *not* tautologous, that refutes cross-axiom models in the completeness conjectures.