

Refutation of the Hrushovski construction, to confirm Lachlan and Zil'ber

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Abstract: A condition for the Hrushovski construction is *not* tautologous, refuting it. This also denies alleged refutations using it, namely, to confirm the Lachlan conjecture and Zil'ber conjecture. The construction forms a *non* tautologous fragment of the universal logic $V\mathbb{L}4$.

We assume the method and apparatus of Meth8/ $V\mathbb{L}4$ with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET \sim Not, \neg ; + Or, \vee, \cup, \sqcup ; - Not Or; & And, $\wedge, \cap, \square, \cdot, \otimes$; \ Not And;
 $>$ Imply, greater than, $\rightarrow, \Rightarrow, \mapsto, >, \supset, \Rightarrow$; $<$ Not Imply, less than, $\in, <, \subset, \neq, \neq, \leftarrow, \lesssim$;
 $=$ Equivalent, $\equiv, :=, \Leftrightarrow, \leftrightarrow, \stackrel{\Delta}{\sim}, \approx, \simeq$; @ Not Equivalent, \neq, \oplus ;
 $\%$ possibility, for one or some, \exists, \diamond, M ; # necessity, for every or all, \forall, \square, L ;
 $(z=z)$ **T** as tautology, \top , ordinal 3; $(z@z)$ **F** as contradiction, $\emptyset, \text{Null}, \perp$, zero;
 $(\%z>\#z)$ **N** as non-contingency, Δ , ordinal 1; $(\%z<\#z)$ **C** as contingency, ∇ , ordinal 2;
 $\sim(y < x)$ ($x \leq y$), ($x \subseteq y$), ($x \sqsubseteq y$); $(A=B)$ $(A \sim B)$.
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Hrushovski_construction

The construction

Let L be a finite relational language. Fix \mathbf{C} a class of *finite* L -structures which are closed under isomorphisms and substructures. We want to strengthen the notion of substructure; let \leq be a relation on pairs from \mathbf{C} satisfying:

$$A \subseteq B \subseteq C \text{ and } A \leq C \text{ implies } A \leq B \quad (1.1)$$

LET $p, q, r: A, B, C$.

$$\sim(r < \sim(q < p)) \& (\sim(r < p) > \sim(q < p)); \quad \mathbf{TTF\ TTF\ TTF\ TTF} \quad (1.2)$$

$$\sim(C < \sim(B < A)) \& (\sim(C < A) > \sim(B < A)); \quad \mathbf{TTTT\ NTNT\ CCTT\ FCNT} \quad (1.3)$$

Eqs. 1.2 and 1.3 as rendered are *not* tautologous, refuting the Hrushovski construction. This denies refutations using it, namely, to confirm the Lachlan conjecture and Zil'ber conjecture.