Refutation of Leibnitz' indiscernibles

Abstract: Four equations defining Leibnitz' law of indiscernibles are *not* tautologous, hence refuting it. What follows is that these theories are also refuted: Ramsey cardinals; Erdös cardinals; and Zero sharp. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Indiscernibles

If a, b, and c are distinct and {a, b, c} is a set of indiscernibles, then, for example, for each binary formula β, we must have

[β(a,b)∧β(b,a)∧β(a,c)∧β(c,a)∧β(b,c)∧β(cb)] ∨
[¬β(a,b)∧¬β(b,a)∧¬β(a,c)∧¬β(c,a)∧¬β(b,c)∧¬β(cb)] (1.1)

LET p; q; r; s: β or φ or F; a or x; b or y; c.

((((p&(q&r))&(p&(r&q)))&((p&(q&s))&(p&(s&q))))&((p&(r&s))&
(p&(s&r))))
+
((((~p&(q&r))&(~p&(r&q)))&((~p&(q&s))&(~p&(s&q))))&((~p&(r&s))&
(~p&(s&r)))) ; **FFFF FFFF FFFF FF**TT (1.2)

Remark 1.2: Eq. 1.2 collapses logically to:

(p&((q&r)&s))+(~p&((q&r)&s)) ;
FFFF FFFF FFFF FFTT (1.3)

In some contexts one considers the more general notion of *order-indiscernibles*, and the term *sequence of indiscernibles* often refers implicitly to this weaker notion. In our example of binary formulas, to say that the triple (a, b, c) of distinct elements is a sequence of indiscernibles implies

([φ(a,b)∧ φ(a,c)∧ φ(b,c)]∨[¬φ(a,b)∧¬φ(a,c)∧¬φ(b,c)])∧
([φ(b,a)∧ φ(c,a)∧ φ(c,b)]∨[¬φ(b,a)∧¬φ(c,a)∧¬φ(c,b)]) (2.1)

((((p&(q&r))&(p&(q&s)))&(p&(r&s)))+(((~p&(q&r))&(~p&(q&s)))&(~p&(r&s))))
&
((((p&(r&q))&(p&(s&q)))&(p&(s&r)))+(((~p&(r&q))&(~p&(s&q)))&(~p&(s&r))) ;

FFFF FFFF FFFF FFTT (2.2)

Remark 2.2: Eq. 2.2 collapses logically to 1.3. (2.3)

Eqs. 1.2, 1.3, 2.2, and 2.3 as rendered are *not* tautologous, refuting Leibnitz' law of indiscernibles. What follows is that the these theories are also refuted: Ramsey cardinals; Erdös cardinals; and Zero sharp.