Refutation of completeness of temporal logics over infinite intervals

Abstract: We evaluate three pairs of equations for interval temporal logics [ITL]. None is tautologous, refuting completeness of temporal logics for finite *and* infinite intervals. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
\> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Wang, H.; Xu, Q. (2004). Completeness of temporal logics over infinite intervals.
fst.um.edu.mo/en/staff/documents/fstqx/CompletenessInfinite.pdf

Abstract Interval temporal logics over infinite intervals are studied. First, the ordinary possible worlds models are extended to infinite possible world models. Accordingly, an axiomatic system is proposed and it has been proved complete. Secondly, infinite intervals are included in a logic over abstract intervals. A corresponding axiomatic system is given and proven to be complete also.

2 Interval temporal logic [ITL] **over finite intervals**
2.4 System S'
Axiomatic system S' concentrates on reasoning about intervals rather than just possible worlds.
Temporal and duration domain A duration domain D is a non-empty set equipped with a binary operation + and at least one element 0 which satisfy the conditions D1–D5 below

D5 (∃z)(x + z = y ∨ y + z = x), (2.4.5.1.1)
(∃z)(z + x = y ∨ z + y = x). (2.4.5.2.1)

LET p, q, r: x, y, z.

((%r+p)=q)+((%r+q)=p) ; NCCT **F**TTT NCCT **F**TTT (2.4.5.1.2)
((p+%r)=q)+((q+%r)=p) ; NCCT **F**TTT NCCT **F**TTT (2.4.5.1.2)

4.1 Infinite interval models
Duration domain Let D be an algebra with a binary operation + and two distinct constants 0 and ∞. D is called a duration domain if the algebra satisfies the following conditions

(6) There exists z such that x+z=y or y+z=x, and (4.1.6.1.1)
there exists z such that z+x=y or z+y=x. (4.1.6.2.1)

((%r+p)=q)+((%r+q)=p) ;   `NCCT` **`F`**`TTT NCCT` **`F`**`TTT`   (4.1.6.1.2)

((p+%r)=q)+((q+%r)=p) ;   `NCCT` **`F`**`TTT NCCT` **`F`**`TTT`   (4.1.6.1.2)

4.2 System S'$_\infty$

System S'$_\infty$ for infinite interval models is obtained by adding the following new axioms to S$_\infty$ [about Duration domain]:

D6 $(\exists x)(x + z = y \lor y + z = x)$,   (4.2.6.1.1)

$(\exists x)(z + x = y \lor z + y = x)$;   (4.2.6.2.1)

((%p+r)=q)+((q+r)=%p) ;   `N`**`F`**`CT CTTT N`**`F`**`CT CTTT`   (4.2.6.1.2)

((r+%p)=q)+((q+r)=%p) ;   `N`**`F`**`CT CTTT N`**`F`**`CT CTTT`   (4.2.6.2.2)

Eqs. 2.4.5, 4.1.6, and 4.2.6 are *not* tautologous. This refutes the conjectures of completeness of temporal logics over finite *and* infinite intervals.