Refutation of the variety of distributive bilattices

Abstract: The example stated of a distributive bilattice is not tautologous. This refutes that variety and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moraschini, T. (2019). A study of truth predicates in matrix semantics.
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4. Semilattice-based examples

In this section we review a family of natural examples of logics whose truth sets are almost parametrically equationally, but not equationally, definable. In the light of Corollary 3.10 we know that all these examples need to be purely inferential.

Example 4.4 (Distributive Bilattices).
An algebra A = ⟨A, ∧, ∨, ⊗, ⊕, ¬⟩ is a bilattice if ⟨A, ∧, ∨, ⊗, ⊕⟩ is a pre-bilattice such that ¬¬a = a and [for every a, b ∈ A ..]

a≤b⇒(¬b≤¬a and ¬a⊑¬b) (4.4.1.1)

LET p, q: a, b.

~(q<p)>(~(~p<~q)&~(~q<~p)) ;
T**F**TT T**F**TT T**F**TT T**F**TT (4.4.1.2)

Eq. 4.4.1.2 as rendered is *not* tautologous, refuting distributive bilattices.