**Refutation of varieties of positive modal logic (PML)**



**Abstract:** Four definitions of positive modal logic (PML) are *no*t tautologous. This refutes positive modal algebra (PML) on the bounded distributive lattice and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~( y < x) ( x ≤ y), ( x ⊆ y), ( x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moraschini, T. (2019). Varieties of positive modal algebras and structural completeness. arxiv.org/pdf/1908.01659.pdf moraschini@cs.cas.cz

**3. Algebras and frames**

**Definition 3.1.** A *positive modal algebra* is a structure $A=\langle A,\wedge,\vee,\Box,\Diamond,0,1\rangle$ where $\langle A,\wedge,\vee,0,1\rangle$ is a bounded distributive lattice such that [for every $a,b\in A$]

$\Box 1=1$ and (3.1.1.1)

LET p, q, s: a, b, s.

#(s=s)=(s=s) ; NNNN NNNN NNNN NNNN (3.1.1.2)

**Remark 3.1.1.2:** Ordinal 1 as N (%s>#s) produces a theorem, but the author means T for (s=s).

$\Diamond 0=0$ and (3.1.2.1)

%(s@s)=(s@s) ; NNNN NNNN NNNN NNNN (3.1.2.2)

**Remark 3.1.2.2:** Ordinal 2 as C (%s<#s) produces a theorem, but the author means **F** for (s@s).

$\Box a\wedge\Diamond b\leq\Diamond(a\wedge b)$ (3.1.5.1)

~(%(p&q)<(#p&%q))=(s=s) ; NNNN NNNN NNNN NNNN (3.1.5.2)

$$\Box(a \vee b) \leq \Box a \vee \Diamond b \tag{3.1.6.1}$$

~((#p+%q)<#(p+q))=(s=s) ; NNNN NNNN NNNN NNNN (3.1.6.2)

Eqs. 3.1.1.2, ..2.2, ..5.2, and ..6.2 are *no*t tautologous. This refutes positive modal algebra (PML) on the bounded distributive lattice.