Fermat's Last Theorem (excluding the case of n=2^t). Unified method

In Memory of my MOTHER

Theorem. The equation

0°) $X^m=Z^m-Y^m$, where the number m (=tn) has a prime co-factor $n>2$, has no solution in natural numbers.

The Fundamentals of the Theory of Prime Numbers and the Fermat's Equality 0°:

All calculations are done with numbers in base n, a prime number greater than 2. The simplest proofs and calculations from the school program are omitted.

Notations.

A', A'', $A_{(k)}$ – the first, the second, the k-th digit from the end of the number A;

$A_{[k]}$ – is the k-digit ending of the number A (i.e. $A_{[k]}=A \bmod n^k$);

With the replacement $X^t=A^n; Z^t=B^n; Y^t=C^n$ the equality 0° comes down to the equality

1°) ($D=...$) $A^n+B^n-C^n=0$, whence, using the decomposition formulas:

2°) ($D=...$) $(C-B)P+(C-A)Q-(A+B)R=0$.

3°) After dividing the equality 1° by T^n, where T is the greatest common divisor of the numbers A, B, C, the numbers A, B, C with the new values become in pairs coprime integers.

4°) **Theorem**. With $A' \neq 0$, $B' \neq 0$, $C' \neq 0$ the numbers in the pairs $(C-B, P)$; $(C-A, Q)$; $(A+B, R)$ in the equality 2° are coprime integers. The truth of the statement follows from the representation of the number P (similarly of the numbers Q and R) in its decomposition formula in the form

4a°) $P=S(C-B)^2+nC^{(n-1)/2}B^{(n-1)/2}$, where $C-B$, C and B are coprime integers.

4b°) **Consequence of 4° and 4a°**. If $A' = n^kA°$, where $A°' \neq 0$, *then* $P' = 0$, $P'' \neq 0$, $C-B=a^nn^{kn-1}$;

4c°) **Consequence of 4°**. If $(ABC)' \neq 0$, then $C-B=a^n$; $C-A=b^n$; $A+B=c^n$; $P=p^n$; $Q=q^n$; $R=r^n$.

5°) If $A \neq 0$, then $(A^{n-1})' = 1$ [Fermat's little theorem].

6°) If $(ABC)' \neq 0$, then [consequence of 1°, 2° and 5°] $P'=Q'=R'=1$, whence

7°) $P_{[2]}=Q_{[2]}=R_{[2]}=01$ [the Newton's binomial fot the number $A=(A°n+1)^n$],

8°) Therefore [4c° and the Newton's binomial], $p'=q'=r'=1$.

9°) Therefore [2° and 7°], if $(ABC)'\neq 0$, then $(A+B-C)_{[2]}=0$.

10°) Therefore [9°], $(A+B-C)'=0$.

11°) Therefore [9° and 10°], $(A+B-C)''$ is equal either to 0, or to $n-1$.

12°) **<u>Theorem</u>.** All n digits $(gt)'$, where $0<g<n$ and $t=1, 2, ... n$, are different.

13°) **<u>Consequence</u>.** For a given digit $g\neq 0$, such a digit t exists that $(gt)' = 2$

13a°) If $A' \neq 0$ and $A_{[2]}=A^n_{[2]}$, then for a given $A_{[t]}$, such a number g^{nn} exists that $(Ag^{nn})_{[t]}=1$.

14°) **<u>Theorem</u>.** The sum $S=1^n+2^n+...(n-1)^n$ ends by 00 and the digit S''' is equal to $(n-1)/2$.

15°) **<u>Consequence</u>.** If $(ABC)'\neq 0$ and $(A'^n+B'^n-C'^n)_{[2]}=0$, then all $E''' = (A'^n+B'^n-C'^n)''' > 0$ [otherwise the sum $[(A'^n+B'^n-C'^n)ti^n]''' = 0$ $(i=1, 2, \dots n-1)$, and not $(n-1)/2$].

16°) The digit $A^n_{(k+1)}$ is uniquely defined by the ending $A_{[k]}$ and therefore, the ending $A^n_{[2]}$ does not depend on the digit A''. This fact follows from the rewriting of the number A into the form $A=dn+A'$ and the decomposition of the binomial $A^n=(dn+A')^n$.

17°) If $A=A^{\circ n}n^{2n}+1$, then $(A^{\circ n}n^{2n}+1)^n=...+[(n-1)/2]A^{\circ 2}n^{4n+1}+A^{\circ n}n^{2n+1}+1$ [cf. the Newton's binomial].

18°) If $A^n=Xn^{4n+1}+A°'n^{2n+1}+1$, where $A^{\circ n}<n^n$, then $A=...+A°n^{2n}+1$ [17°].

19°) In the equality 3° the number $D=E+F$, where $E = A'^n+B'^n-C'^n$ and $F=(A''+B''-C'')n^2+Gn^3$.

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The Proof of FLT. First Case [(ABC)'≠0]

Using multiplication of the equality 3° by some number g^{nnn} [wherein the properties of 4b°-13a° persist!] we transform the digit *E'''* into *2* [15° and 13°].

We can see in the Newton's binomials for the numbers *A, B, C* [19°] , that in order to transform that digit to zero, the digit *(A''+B''-C'')'* must be equal to *n-2*. However, it is equal to either to 0, or to n-1 [11°], and thus the equality 1° is not verified on the third digit.

Second Case [for example A'=0, but (BC)'≠0]

Let's assume that for co-prime natural numbers *A* [$A=n^kA°$], *B* and *C*

20°) $A^n=C^n-B^n$ and $C^n-B^n=(C-B)P$, where $(C-B)_{[kn-1]}=0$, $P=P°n$, $A^n=n^{kn}A^{°n}$ [4b°].

Using multiplication of the equality 20° by the appropriate number g^{nnn} let's transform the ending of the number *B* having the length of *3kn* digits, into *1* [13a°]. Whereupon [4b°] in the new 20°

21°) $A=an^k$, $C=cn^{kn-1}+1$, $B=...n^{3kn}+1$; $A^n=a^nn^{kn}$, $C^n=C°n^{kn}+1=...cn^{kn}+1$, $Bn=...n^{3kn+1}+1$.

After that we will leave in the numbers *A°*, *B*, *C* only the last digits *a*, *1*, *1* and will calculate the (*3kn-2*)-digits endings of the numbers A^n and C^n (wherein $B_{[3kn]}=1$):

22°) $a \Rightarrow a^n_{[n]}$; $\Rightarrow c_{[n]}=a^n_{[n]}$, $\Rightarrow$ [21°] $C^n=...+c_{[n]}n^{kn}+1=...+a^n_{[n]}n^{kn}+1$, $\Rightarrow C$ [18°]:

23°) $C=(...+c_{[n]}n^{kn}+1)^{1/n}=...+a^n_{[n]}n^{kn-1}+1 \Rightarrow C^n$ [17°]:

24°) $C^n=...[(n-1)(n-2)/6]a^{3n}n^{3kn-2}+[(n-1)/2]a^{2n}n^{2kn-1}+a^nn^{kn}+1, \Rightarrow A^n$ [21°]:

25°) $A^n=...[(n-1)(n-2)/6]a^{3n}n^{3kn-2}+[(n-1)/2]a^{2n}n^{2kn-1}+a^nn^{kn}=$

$=a^nn^{kn}\{...[(n-1)(n-2)/6]a^{2n}n^{2kn-2}+[(n-1)/2]a^nn^{kn-1}+1\}$, where the expression in braces is the n-th degree [18°] of the number $...[(n-1)/2]a^nn^{kn-2}+1$, that is [17°]:

26°) $A^n=a^nn^{kn}\{...[(n-1)(n-1)(n-1)/8]a^{2n}n^{2kn-3}+[(n-1)/2]a^nn^{kn-1}+1$, or

26a°) $A^n=...[(n-1)(n-1)(n-1)/8]a^{3n}n^{3kn-3}+[(n-1)/2]a^{2n}n^{2kn-1}+a^nn^{kn}$.

Now, if we compare 24° and 26a°, we are having a contradiction in the equality 21 on the digit (*3kn-2*): in the 26a° it is DIFFERENT from zero, yet in the 24° it is ZERO!

At the same time, as you can see in 24° and 26a°, the restoration of all previous digits in number *A*° can not correct this contradiction, because it is only defined by the digit *a'*.

Thus The Fermat's Last Theorem is verified.

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