Refutation of symmetry breaking, Boolean skeletons, ensemble technique, and SMT solver

Abstract: The SMT problem as stated is not a theorem, and the derivation of Boolean skeletons, while equivalent, are not the SMT problem. This refutes the symmetry breaking technique and also the attendant ensemble technique. What follows is the SMT solver is refuted, forming a non tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Let:
\[
\neg \text{ Not, } \neg; \quad \lor \text{ Or, } \lor, \lor; \quad \land \text{ And, } \land, \land; \quad \setminus \text{ Not Or; } \setminus \land \text{ And; }
\]

\]

\[
> \text{ Imply, greater than, } \rightarrow, \Rightarrow, \implies, >, \supseteq, \supseteq; \quad < \text{ Not Imply, less than, } \in, \subset, \preceq, \preceq, \subseteq; \quad = \text{ Equivalent, } \equiv, \equiv, \equiv, \equiv;
\]

\[
\% \text{ possibility, for one or some, } \exists, \bigcirc, M; \quad \# \text{ necessity, for every or all, } \forall, \Box, L;
\]

\[
(z=z) \text{ T as tautology, } T, \text{ ordinal 3}; \quad (z@z) \text{ F as contradiction, } \emptyset, \text{ Null, } \perp,
\]

\[
(%z>=%z) \text{ N as non-contingency, } \Delta, \text{ ordinal 1}; \quad (%z<=%z) \text{ C as contingency, } \nabla, \text{ ordinal 2};
\]

\[
\prec (y < x) (x \leq y), (x \leq y), (x \leq y) ; \quad (A=B) (A\neq B).
\]

Note for clarity, we usually distribute quantifiers onto each designated variable.


1 Introduction
Satisfiability Modulo Theories (SMT) is a decision problem for logical first order formulas combined with operations defined over additional constructs such as integers, reals, arrays, and uninterpreted functions... Symmetry breaking... has been an effective technique for improving the efficiency of propositional logic solvers for a long time. It involves identifying variable permutations (known as symmetry permutations) applying which does not alter the theory, and then using them to add constraints to the problem without changing it’s satisfiability and thereby reducing the search space.

3.1 Symmetry Breaking Technique
SMT problem $\Omega$  
\[
(x<8) \land (y<8) \land ((x+y<10) \lor (x+y>3))
\]

Let
\[
P, Q, R, S: \quad T, Q, R, S.
\]

\[
((x<q) \land (y<q)) \land ((x+(y<r)) \land (x+(y>p))) ;
\]

\[
\text{FFFF FFFF FFFF FFFF ( 48) ;}
\]

\[
\text{TTFF TTFF TTFF TTFF ( 16) (3.1.1.2)
}\]

Boolean skeleton $\Psi$  
\[
Q \land R \land (S \lor T)
\]

\[
(q\land r) \land (s+p) ;
\]

\[
\text{FFFF FFFF FFFF FFFFF (3.1.2.2)
}\]

Constraints set ...
Symmetry permut. ...
SBP added …
New skeleton $\Psi' \quad Q \land R \land (S \lor T) \land (\neg Q \lor R)$ \hspace{1cm} (3.1.6.1)

$$(q \land r) \land ((s \lor p) \land (\neg q \lor r))$$ \hspace{1cm} F F F F T T

Eqs. 3.1.1.2, 3.1.2.2, and 3.1.6.2 are not tautologous. Eqs. 3.1.2.2 and 3.16.2 are the same. The SMT problem is not a theorem, and the Boolean skeletons, while equivalent, are not the SMT problem. This refutes the symmetry breaking technique and also the attendant ensemble technique. What follows is the SMT solver is also refuted.