Refutation of classification of finitary, algebraizable logics as undecidable in Hilbert calculi

Abstract: Two axioms as tested are *no*t tautologous, to refute the Hilbert calculus as claimed. This refutes the conjecture of classification of finitary, algebraizable logic as undecidable in Hilbert calculi. That also disallows a follow-on article. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moraschini, T. (2019). A computational glimpse at the Leibniz and Frege hierarchies. arxiv.org/pdf/1908.00922.pdf

Abstract … algebraic logic (AAL for short) is a field that studies uniformly propositional logics.. . One of its main achievements is the development of the so-called Leibniz and Frege hierarchies in which propositional logics are classified according to two different criteria. More precisely, the Leibniz hierarchy provides a taxonomy that classifies propositional systems accordingly to the way their notions of logical equivalence and of truth can be defined. Roughly speaking, the location of a logic inside the Leibniz hierarchy reflects the strength of the relation that it enjoys with its algebraic counterpart. In this sense, the Leibniz hierarchy revealed to be a useful framework where to express general transfer theorems between metalogical and algebraic properties. This is the case for example for superintuitionistic logics… On the other hand, the Frege hierarchy offers a classification of logics according to general replacement principles. Remarkably, some of these replacement properties can be formulated semantically by asking that the different elements in a model of the logic are separated by a deductive filter. This is what happens for example in superintuitionistic logics, whose algebraic semantics is given by varieties of Heyting algebras where logical filters are just lattice filters. The aim of this paper is to investigate the computational aspects of the problem of classifying syntactically presented logics in the Leibniz and Frege hierarchies. More precisely, we will consider the following problem

> Let K be a level of the Leibniz (resp. Frege) hierarchy. Is it possible to decide whether the logic of a given finite consistent Hilbert calculus in a finite language belongs to K?

It turns out that in general the answer is negative both for the Leibniz and the Frege hierarchies. … Remarkably, our proof shows that this classification problem remains undecidable even if we restrict our attention to Hilbert calculi that determine a finitary algebraizable logic (Theorem 5.3).

5. The classification problem in the Frege hierarchy

Definition 5.1. … with two new connectives □ and →, respectively unary and binary. *L*(*a*, *b*) is the logic in the language *L* axiomatized by the following Hilbert calculus … for every formula φ of the following … :

$$\varphi_4 := x \rightarrow (x \rightarrow \Box x) \tag{5.1.4.1}$$

LET p: x.

p>(p>#p); TNTN TNTN TNTN TNTN (5.1.4.2)

$$\varphi_6 := (\Box x \rightarrow x) \rightarrow ((x \rightarrow \Box x) \rightarrow x) \tag{5.1.6.1}$$

(#p>p)>((p>#p)>p) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (5.1.6.2)

Eqs. 5.1.4.2 and 5.1.6.2 as rendered are *not* tautologous. This means two axioms refute the Hilbert calculus as used. This also refutes the conjecture of classification of finitary, algebraizable logic as undecidable in Hilbert calculi and further disallows the follow-on article: Moraschini, T. (2019). On the complexity of the Leibniz hierarchy. arxiv.org/pdf/1908.00924.pdf