Refutation of Heyting algebra

Abstract: We evaluate the seminal equation for Heyting algebra of a∧b≤c⇔a≤b→c. It is *not* tautologous, hence refuting Heyting algebra as stated, and forming another *non* tautologous fragment of Heyting algebra in the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moraschini, T.; Wannenburg, J.J. (2019) Epimorphism surjectivity in varieties of Heyting algebras. arxiv.org/pdf/1908.00287.pdf

2. Esakia duality

A *Heyting algebra* is an algebra ***A*** = ⟨*A*; ∧, ∨, →, 0, 1⟩ which comprises a bounded lattice ⟨*A*; ∧, ∨, 0, 1⟩, and a binary operation → such that for every *a*, *b*, *c* ∈ *A*,

$$a \wedge b \leq c \Leftrightarrow a \leq b \rightarrow c. \quad (2.1.1)$$

LET p, q, r: *a*, *b*, *c*.

(p&~(q<r))=(~(q<p)>r) ; TFFT FTFT TFFT FTFT (2.1.2)

It follows that Heyting algebras are distributive lattices. Remarkably, a Heyting algebra is uniquely determined by its lattice reduct. The class of all Heyting algebras forms a variety, ... HA.

Eq. 2.1.2 as rendered is *not* tautologous, hence refuting Heyting algebra as stated.