Refutation of Lӧwner (Loewner) order and quantum temporal logic

Abstract: We evaluate the Löwner order ⊑ for positive definite as the basis for quantum temporal logic (QTL). The operator is *not* tautologous. We also evaluate the semantics for QTL in three operators, also *not* tautologous. These form *non* tautologous fragments for both in the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Yu, N. (2019). Quantum temporal logic: from Birkhoff and von Neumann to Pnueli. arxiv.org/pdf/1908.00158.pdf

2. Preliminaries

A Hermitian operator A is *positive semidefinite* (resp., *positive definite*) if for all vectors |ψi ∈ H, hψ| A|ψi ≥ 0 (resp., > 0). This gives rise to the *Löwner order* ⊑ among operators:

A ⊑ B if B−A is positive semidefinite, A ⊏ B if B−A is positive definite. (2.1.1.1, 2.1.2.1)

((B-A)>(C@C))>(A<B) ; TTTT NNNN CCCC **FFFF** (2.1.2.2)

Remark 2.1.2.2: Eq. 2.1.2.2 as rendered is *not* tautologous, hence refuting the Löwner order for positive definite.

4.2. Semantics for QTL [quantum temporal logic]

[2] For p, q ∈ AP, p∨q is the union of subspaces p and q, p∨q is not always in AP. (4.2.5.1.2)

LET p, q, r, s: p, q, A, P

((p&q)<(r&s))>~(#((p+q)<(r&s))=(s=s)) ;
TTTC TTTC TTTC TTTT (4.2.5.1.2)

The additional logical operators are defined as follows:

φ → ψ ≡ L(φ) ⊂ L(ψ) (4.2.10.1)

LET p, q, r, s: φ, L, r, ψ.

$(p>s)=((q\&p)<(q\&s))$; **F**T**FF** **F**T**FF** **FFFF** **FFFF** (4.2.10.2)

$$\varphi \leftrightarrow \psi \equiv (\psi \rightarrow \varphi) \wedge (\varphi \rightarrow \varphi) \qquad (4.2.11.1)$$

$(p=s)=((s>p)\&(p>p))$; T**F**T**F** T**F**T**F** TTTT TTTT (4.2.11.2)

Remark 4.2: Eqs. 4.2.5.1.2, 4.2.10.2, and 4.2.11.2 are *no*t tautologous. This refutes the semantics for QTL, and hence QTL.