Refutation of the wellfoundedness of the multiset order

Abstract: The two equations for an inductive proof of the wellfoundedness of the multiset order are *not* tautologous, and form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Nipkow, T.; Buchholz, W. (1998). An inductive proof of the wellfoundedness of the multiset order. www21.in.tum.de/~nipkow/misc/multiset.ps [partial file]

Given a binary relation < on a set S, the subset of S called the *well-founded part* of S w.r.t < is defined inductively as follows ..:

$$\frac{\forall y<x.y\in W}{x\in W} \tag{1.1.1}$$

LET p, q, r, s: P, w, x, y.

(#s<(r&(s<q)))>(r<q) ; TTTT TTTT CCCC TTCC (1.1.2)

The corresponding induction principle easily yields the principle of *well-founded part induction*:

$$\frac{\forall x\in W.(\forall y<x.P(y))\Rightarrow P(x)}{\forall x\in W.P(x)} \tag{1.2.1}$$

(((#r<q)&(#s<(r&(p&s))))>(p&r))>(#r<(q&(p&r))) ;
FFFF NNN**F** **FFFF** NNN**F** (1.2.2)

Eqs. 1.1.2 and 1.2.2 as rendered are *not* tautologous. This refutes the conjecture of an inductive proof of the wellfoundedness of the multiset order.