

Refutation of the Church-Rosser theorem

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Abstract: The Church-Rosser theorem evaluates as *not* tautologous, hence forming a *non* tautologous fragment of the universal $\forall\mathcal{L}4$.

We assume the method and apparatus of Meth8/ $\forall\mathcal{L}4$ with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET \sim Not, \neg ; + Or, \vee , \cup , \sqcup ; - Not Or; & And, \wedge , \cap , \sqcap , \cdot , \otimes ; \ Not And;
 $>$ Imply, greater than, \rightarrow , \Rightarrow , \mapsto , $>$, \supset , \rightarrow ; $<$ Not Imply, less than, \in , $<$, \subset , \prec , \neq , \ll , \lesssim ;
 $=$ Equivalent, \equiv , $:=$, \Leftrightarrow , \leftrightarrow , $\hat{=}$, \approx , \cong ; @ Not Equivalent, \neq , \oplus ;
 $\%$ possibility, for one or some, \exists , \diamond , \mathbb{M} ; # necessity, for every or all, \forall , \square , \mathbb{L} ;
 $(z=z)$ **T** as tautology, \top , ordinal 3; $(z@z)$ **F** as contradiction, \emptyset , Null, \perp , zero;
 $(\%z\>\#z)$ **N** as non-contingency, Δ , ordinal 1; $(\%z\<\#z)$ **C** as contingency, ∇ , ordinal 2;
 $\sim(y < x)$ ($x \leq y$), ($x \subseteq y$), ($x \sqsubseteq y$); $(A=B)$ $(A\sim B)$.

Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Church-Rosser_theorem

If term a can be reduced to both b and c , then there must be a further term d (possibly equal to either b or c) to which both b and c can be reduced. (1.1)

$$\text{LET } p, q, r, s: \quad a, b, c, d.$$

$$(p \rightarrow (q \& r)) \rightarrow (s = (q + r)) \rightarrow ((q \& r) \rightarrow s);$$

TTTT TTNN TTTT TTTT

(1.2)

Remark 1.2: Eq. 1.2 may also be rendered as

$$(s = (q + r)) \rightarrow ((p \rightarrow (q \& r)) \rightarrow ((q \& r) \rightarrow s)) \text{ with the same truth table result.} \quad (1.3)$$

Eqs. 1.2 and 1.3 are *not* tautologous, hence refuting the Church-Rosser theorem.