Refutation of completeness for inclusion and equivalence of universality

Abstract: We evaluate a formula for inclusion and equivalence of universality. It is *not* tautologous, refuting the conjecture of completeness, and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Masopust, T.; Krötzsch, M. (2019). Partially ordered automata and piecewise testability. arxiv.org/pdf/1907.13115.pdf

Abstract: Universality is the question whether a system recognizes all words over its alphabet. Complexity of deciding universality provides lower bounds for other problems, including inclusion and equivalence of systems behaviors. We study the complexity of universality for a class of nondeterministic finite automata, models as expressive as boolean combinations of existential first-order sentences. **Conclusion:** [W]e obtained PSpace-completeness for several restricted types ... for problems including inclusion, equivalence, and (k-)piecewise testability.

7. Inclusion and equivalence: A consequence of the complexity of universality is the worst-case lower-bound complexity for the inclusion and equivalence problems. These problems are of interest, e.g., in optimization. … Although equivalence means two inclusions, complexities of these two problems may differ significantly, e.g., inclusion is undecidable for deterministic context-free languages.. while equivalence is decidable.. . Since universality can be expressed as the inclusion Σ∗ ⊆ L or the equivalence Σ∗ = L, we immediately obtain the hardness results for inclusion and equivalence from the results for universality. Therefore, it remains to show memberships of our results … Let A be an automaton of any of the considered types ... depending on the type of B. We assume that both automata are over the same alphabet specified by B. If B is a DFA, then

L(A) ⊆ L(B) if and only if L(A) ∩ L(B) = ∅, (7.1.1)

LET p, q, r, s: A, B, L, s.

(((r&p)&(r&~q))=(s@s))>~((r&q)<(r&p)) ;
TTTT TT**F**T TTTT TT**F**T (7.1.2)

Remark 7.1.2: Eq. 7.1.2 as rendered is *not* tautologous. This refutes the hardness results for inclusion and equivalence from the results for universality, meaning the conjecture is not complete.