

Refutation of Turing's halting problem as logically unsolvable

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Abstract: We confirm the halting conjecture as tautologous and hence refute the halting problem as unsolvable. What follows is that first order logic is decidable. This proof was made possible by the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬; + Or, ∨, ∪, ∪; - Not Or; & And, ∧, ∩, ∩, ·, ⊗; \ Not And;
 > Imply, greater than, →, ⇒, ⇨, >, ⊃, →; < Not Imply, less than, ∈, <, ⊂, ≠, ≠, ≠, ≲;
 = Equivalent, ≡, :=, ⇔, ↔, ≐, ≈, ≈; @ Not Equivalent, ≠, ⊕;
 % possibility, for one or some, ∃, ∃, M; # necessity, for every or all, ∀, □, L;
 (z=z) T as tautology, T, ordinal 3; (z@z) **F** as contradiction, ∅, Null, ⊥, zero;
 (%z>#z) **N** as non-contingency, Δ, ordinal 1; (%z<#z) **C** as contingency, ∇, ordinal 2;
 ~(y < x) (x ≤ y), (x ⊆ y), (x ⊆ y); (A=B) (A~B).
 Note for clarity, we usually distribute quantifiers onto each designated variable.

We cast Turing's halting problem as follows.

If stop is contradictory and a program step is less than the program executing it, then:
 if the predictor indicates stop, then
 if the program is no longer executing then the program step is stopped
 or
 if the predictor indicates not to stop, then
 if the program is executing then the program step is not stopped. (1.1)

LET p, q, r, s, t:
 P prediction; Q step within the program loop; loop; stop switch; instant experiment.

$$(((s=(s@_s))\&(q<(r>r)))>(((p>s)>((r<r)>(q>s)))+(p>\sim s)>((r>r)>(q>\sim s))))=(s=s);$$

TTTT TTTT TTTT TTTT

(1.2)

The iteration aspect of the problem is handled as follows in the form of (T=T)>(T>T).

If the instant experiment is equivalent to the halting problem in Eq. 1.1, then
 if the halting problem in Eq. 1.1, then
 the instant experiment. (2.1)

$$(t=(((s=(s@_s))\&(q<(r>r)))>(((p>s)>((r<r)>(q>s)))+(p>\sim s)>((r>r)>(q>\sim s))))>$$

$$(((s=(s@_s))\&(q<(r>r)))>(((p>s)>((r<r)>(q>s)))+(p>\sim s)>((r>r)>(q>\sim s))))>t;$$

TTTT TTTT TTTT TTTT (128) in 75 steps

(2.2)

Eq.1.2 is tautologous, refuting the halting problem as unsolvable. What follows is that first order logic is decidable.