Trigonometric functions – is there a problem?

Ilija Barukčić

Internist
Horandstrasse, DE-26441 Jever, Germany

Barukcic@t-online.de

Abstract — Trigonometry is an important part of mathematics. In general, a proper understanding of trigonometric functions is a pre-requisite for understanding important topics in physics, architecture, and many branches of engineering. We commonly suppose that the mathematics of trigonometric functions is perfectly true. However, trigonometric functions have a very long and colourful history and the need to take a more precise look on the same is great. This publication provides some evidence that there are circumstances where today understanding of trigonometric functions leads to contradictions.

Keywords — Trigonometric functions, Classical logic, Contradiction

I. Introduction

The knowledge of the true origins of memorable discoveries [1] even if found by accident is of help to recognize the relativity even of mathematical knowledge. In this context, the early studies of triangles can be traced back to the 2nd millennium BC to Egyptian (Rhind Mathematical Papyrus) and Babylonian mathematics. However, a systematic study of trigonometric functions began with the Hellenistic mathematics during the second half of the 2nd century BC. The Hellenistic astronomer Hipparchus of Nicaea (ca. 180 – ca. 125 BC) has been the first to compile a trigonometric table and is known as “the father of trigonometry” [2]. The Hellenistic mathematics reached India [2] where especially Aryabhata (sixth century CE) discovered the sine function. Finally, Aryabhata’s table of sines reached China in 718 AD [3] during the Tang Dynasty. In the following, the studies of trigonometry continued in the Middle Ages by Islamic mathematicians [4] and led to the discovery of all six trigonometric functions. Latin translations of accumulated Arabic knowledge led to trigonometry being adopted in western Europe. In 1342, Levi ben Gershon (1288-1344) [5], known as Gersonides too, worked On Sines, Chords and Arcs [6]. Finally, the western Age of Enlightenment inspired and accelerated the development of modern trigonometry by Jost Bürgi (1552-1632) [7], Henry Briggs (1561-1630) [8], Isaac Newton (1643 - 1727) [9], Roger Cotes (1682-1716) [10], James Stirling (1692-1770) [11], Leonhard Euler (1707-1783) [12] and other too.
II. MATERIAL AND METHODS

A. Definitions

DEFINITION 1. (NUMBER +0)
Let \( c \) denote the speed of light in vacuum, let \( \varepsilon_0 \) denote the electric constant and let \( \mu_0 \) the magnetic constant, let \( i \) denote an imaginary number \[13\]. The number \(+0\) is defined as the expression

\[
+0 \equiv (c^2 \times \varepsilon_0 \times \mu_0) - (c^2 \times \varepsilon_0 \times \mu_0) \\
\equiv +1 - 1 \\
\equiv +i^2 - i^2
\]

while “=” denotes the equals sign or equality sign \[14, 15\] used to indicate equality and “-” \[14, 16, 17\] denotes minus signs used to represent the operations of subtraction and the notions of negative as well and “+” denotes the plus \[16\] signs used to represent the operations of addition and the notions of positive as well.

DEFINITION 2. (NUMBER +1)
Let \( c \) denote the speed of light in vacuum, let \( \varepsilon_0 \) denote the electric constant and let \( \mu_0 \) the magnetic constant, let \( i \) denote an imaginary number \[13\]. The number \(+0\) is defined as the expression

\[
+1 \equiv (c^2 \times \varepsilon_0 \times \mu_0) \equiv -i^2
\]

DEFINITION 3. (THE RIGHT-ANGLED TRINGLE)
A right-angled triangle is a triangle in which one angle is 90-degree angle. Let \( \nu C \) denote the hypotenuse, the side opposite the right angle (side \( \nu C \) in the figure). The sides \( a \) and \( b \) are adjacent to the right angle and are called legs. The side \( b \) adjacent to \( A \) is the side of the triangle which connects \( A \) to the right angle. The third side \( a \) is said to be opposite to \( A \). The following figure may illustrate a right-angled triangle.

![Figure 1. A right-angled triangle](image)

DEFINITION 4. (PYTHAGOREAN THEOREM)
The Pythagorean theorem is defined as

\[
a_t^2 + b_t^2 \equiv \nu C_t^2
\]

DEFINITION 5. (THE NORMALIZATION OF THE PYTHAGOREAN THEOREM)
The normalization \[18, 19\] of the Pythagorean theorem is defined as

\[
\frac{a_t^2}{\nu C_t^2} + \frac{b_t^2}{\nu C_t^2} \equiv +1
\]
**Definition 6. (The Variance Of A Right-Angled Triangle)**

The variance \( \sigma^2 \) of a right-angled triangle \([18, 19]\) is defined as

\[
\sigma_t^2 \equiv \frac{(a_t^2) \times (b_t^2)}{rC_t^2} \times \left( \frac{rC_t^2}{rC_t^2} \right) \tag{5}
\]

**Definition 7. (Sinus Function)**

The sinus function denoted as \( \sin(\text{angle}) \) is a trigonometric function which relate an angle of a right-angled triangle and the ratios of two side lengths. The sinus function is defined as

\[
\sin(\alpha) \equiv \frac{a_t}{rC_t} \tag{6}
\]

and

\[
\sin(\beta) \equiv \frac{b_t}{rC_t} \tag{7}
\]

**Definition 8. (Cosinus Function)**

The sine, the cosine, and the tangent are the most familiar trigonometric functions. The cosine function is defined as

\[
\cosine(\alpha) \equiv \frac{b_t}{rC_t} \tag{8}
\]

and

\[
\cosine(\beta) \equiv \frac{a_t}{rC_t} \tag{9}
\]

**Definition 9. (Tangent Function)**

The tangent function is defined as

\[
tan(\alpha) \equiv \frac{a_t}{b_t} \tag{10}
\]

and as

\[
cotan(\alpha) \equiv \frac{b_t}{a_t} = \frac{1}{tan(\alpha)} \tag{11}
\]

**Definition 10. (Simple Algebraic Values)**

The following table provides an overview [20] about some simplest algebraic values of trigonometric functions.

<table>
<thead>
<tr>
<th>Table 1. Simple algebraic values</th>
<th>0°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sin</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>cosine</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>tan</td>
<td>0</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>
B. Axioms

1) **Axiom I (Lex identitatis. Principium Identitatis. Identity Law)**
   In general, it is
   \[ +1 \equiv +1 \]  \hspace{1cm} (12)
   or the superposition of +0 and +1 as one of the foundations of quantum computing
   \[ +1 \equiv (1 + 0) \times (1 + 0) \times (1 + 0) \times \ldots \times (1 + 0) \]  \hspace{1cm} (13)

2) **Axiom II (Lex contradictionis. Principium contradictionis. Contradiction Law)**
   The (logical) contradiction is expressed mathematically as
   \[ +1 \equiv +0 \]  \hspace{1cm} (14)

3) **Axiom III (Principium negationis)**
   In general, it is
   \[ \frac{+1}{+0} \approx +\infty \]  \hspace{1cm} (15)
III. RESULTS

THEOREM 3.1 (THE CONSEQUENCES OF $\cos(\alpha=0) = 1$)

CLAIM.
Under conditions where $\cos (\alpha = 0) = 1$ it is

$$a_t = 0$$  \hspace{1cm} (16)

PROOF.
In general, according to the rules of trigonometry, the cosine function is defined as

$$\cos(\alpha) \equiv \frac{(b_t)}{(\rho C_t)}$$  \hspace{1cm} (17)

However, it is accepted as correct that $\cos (\alpha = 0) = 1$ [20]. In this case it is

$$\cos(\alpha = 0) \equiv \frac{(b_t)}{(\rho C_t)} = +1$$  \hspace{1cm} (18)

In other words, it is

$$\frac{(b_t)}{(\rho C_t)} = +1$$  \hspace{1cm} (19)

or

$$(b_t) = (\rho C_t)$$  \hspace{1cm} (20)

or

$$(b_t^2) = (\rho C_t^2)$$  \hspace{1cm} (21)

Even under these circumstances, Pythagorean theorem as

$$a_t^2 + b_t^2 \equiv \rho C_t^2$$  \hspace{1cm} (22)

is valid. Rearranging, we obtain

$$a_t^2 + \rho C_t^2 \equiv \rho C_t^2$$  \hspace{1cm} (23)

or

$$a_t^2 \equiv 0$$  \hspace{1cm} (24)

or

$$a_t \equiv 0$$  \hspace{1cm} (25)

QUOD ERAT DEMONSTRANDUM.
THEOREM 3.2 (THE DEFINITION \( \cos(\alpha=0) = 1 \) LEADS TO CONTRADICTIONS)

CLAIM.
The definition \( \cos (\alpha = 0) = 1 \) is logically inconsistent because the same reduces the value of \( b \) only to

\[
b = +1
\]  

(26)

PROOF.
In general, according to the rules of trigonometry it is

\[
\frac{\cos(\alpha)}{\sin(\alpha)} \equiv \frac{b}{\frac{c}{\alpha}} = \frac{b}{a}
\]  

(27)

This relationship is claimed to be valid even if \( \alpha = 0 \). In this case, it is

\[
\frac{\cos(0)}{\sin(0)} = \frac{+1}{+0} = \frac{b}{\frac{c}{\alpha}} = \frac{b}{a}
\]  

(28)

In other words, it is

\[
\frac{+1}{+0} = \frac{b}{a}
\]  

(29)

However, if \( \cos (\alpha = 0) = 1 \) then \( \alpha = 0 \) as proofed by the theorem before. We obtain

\[
\frac{+1}{+0} = \frac{b}{+0}
\]  

(30)

However, whatever the division by 0 may be, \( \cos (\alpha = 0) = 1 \) demands that

\[
b = +1
\]  

(31)

QUOD ERAT DEMONSTRANDUM.

IV. DISCUSSION

Trigonometric functions [21] are widely used in science and our trust into the same appears to be limitless. In one way or another trust is important in science but can be dangerous too. What we risk while trusting without a clear proof of the correctness of something is among other that contradictions make take root in science to such an extent that one definition after the other is necessary to rescue what can be rescued.

In this context, it is important to note, that we cannot rely on trigonometric functions any longer to the extent which is necessary. Especially under conditions, where \( \cos(\alpha = 0) = 1 \), there is a contradiction. The side \( b \), of a right-angled triangle can take values different from 1, especially if \( \alpha = 0 \). However, \( \cos \) demands in this case that \( \cos(\alpha = 0) = 1 \), which leads to is a non-acceptable contradiction.

V. CONCLUSION

It is necessary to review the general validity of the trigonometric functions in detail.
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References

8. Briggs H. Trigonometria Britannica, sive De doctrina triangulorum libri duo : quorum prior continet constructionem canonis sinuum, tangenti&curren#m; & secantium, una cum logarithmis sinuum & ; tangentium ad gradus & ; graduum centesimas & ; ad minuta & ; secunda centesimis respondentia / posterior vero usum sive applicationem canonis in resolutione triangulorum tam planorum quam sphæricorum e geometricis fundamentis petiti, calculo facillimo, eximiiisque compendii exhibet. excudebat Petrus Rammassennius; 1633. doi:10.3931/e-rara-9466.


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