Refutation of Leibniz' identity of indiscernibles and Leo-III theorem prover

Abstract: Leibniz' identity of indiscernibles as ∀x∀y[∀F(Fx↔Fy)→x=y] is *no*t tautologous. Consequently the Leo-III theorem prover for higher-order paramodulated extensional logic is also refuted. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Identity_of_indiscernibles

Identity of indiscernible
For any x and y, if x and y have all the same properties, then x is identical to y:

∀x∀y[∀F(Fx↔Fy)→x=y] (2.1)

LET p, q, r: F, x, y.

((#p&#q)=(#p&#r))>(#q=#r) ; TTCT CTTT TTCT CTTT (2.2)

From: Steen, A.; Benzmüller,C. (2019). Extensional higher-order paramodulation in Leo-III. arxiv.org/pdf/1907.11501.pdf

Abstract: Leo-III is an automated theorem prover for extensional type theory with Henkin semantics and choice. Reasoning with primitive equality is enabled by adapting paramodulation-based proof search to higher-order logic.

3 Extensional higher-order paramodulation

[Fn 4] The Identity of Indiscernibles (also known as Leibniz's law) refers to a principle first formulated by Gottfried Leibniz in the context of theoretical philosophy.. The principle states that if two objects X and Y coincide on every property P, then they are equal, i.e. $\forall X_\tau.\forall Y_\tau.(\forall P_{o\tau}.PX \Leftrightarrow PY) \Rightarrow X=Y$, where "=" denotes the desired equality predicate. Since this principle can easily be formulated in HOL, it is possible to encode equality in higher-order logic without using the primitive equality predicate.

Remark: Eq. 2.2 as rendered is not tautologous, hence refuting Leibniz' identity of indiscernibles, it also refutes the conjecture as title above, including the Leo-III theorem prover for extensional logic.