The Real Numbers are Denumerable in Level Set Theory

by Jim Rock

Abstract. We show Cantor's diagonal argument has an invalid premise. We create a non-hierarchical Level Set Theory by setting $1/\chi_0 = 0$. We prove that the real numbers have the same cardinality as the set of natural numbers, by showing that the power set of the natural numbers has the same cardinality as the natural numbers. This shows there is a one to one mapping from the set of natural numbers to the real numbers, making the real numbers a denumerable set.

The positive integers can be put in one to one correspondence with the terminating decimal fractions in the closed interval **[*0, 1*]**. $0 \to .0,\ 1 \to .1,\ 2 \to .2,\ \ldots,\ 10 \to .01,\ \ldots$ Each terminating decimal is the mirror image reflection through the decimal point of a positive integer. The mapping does not include any repeating decimal fractions. From this mapping the set of all rational numbers would appear to be uncountable.

This shows that attempting to map the real numbers in the closed interval **[*0, 1*]** to the natural numbers, by listing them as infinite decimal fractions is futile. Cantor's diagonal proof that the real numbers are an uncountable set can never even get started.

The problem is in defining the real numbers as the set of all infinite decimal expansions. That's vague. Most non-algebraic real numbers cannot be explicitly referenced. We need a new definition. Let p be an integer. Each real number S_n is the limit of its partial decimal sums:

$$\text{the limit } m \to \infty \quad j = 1 \text{ to } m,\ 0 \le a_j \le 9 \quad \sum p + a_j / 10^j = S_n.$$

For any n the subsets of all the natural numbers between 0 and n can be mapped to $0, 1, 2, 3, \ldots, 2^{n+1} - 1$. By setting $1/\chi_0 = 0$, this process can be extended to the entire set of natural numbers. $|\chi_0| = |2^{\chi_0}|$.

Let $1/\chi_0 = 0$. Then the limit $x \to \infty\ f(x) = \chi_0 \leftrightarrow$ the limit $x \to \infty\ 1/f(x) = 0$.

The **limit** $n \to \infty\ \ n = \chi_0$, and the limit $n \to \infty\ 2^n = \chi_0$. Thus, $|\chi_0| = |2^{\chi_0}|$.

There is no hierarchy of infinites. The limit $n \to \infty\ 2^{2^n} = 2^{2^{\chi_0}} = \chi_0$. $|2^{2^{\chi_0}}| = |\chi_0|$.

Since the power set of the natural numbers has the same cardinality as the set of natural numbers, and the set of real numbers has the same cardinality as the power set of the natural numbers, the set of real numbers has the same cardinality as the set of natural numbers. We can put all the individual real numbers in a set and draw them out one by one: $S_0, S_1, S_2, \ldots$ without replacing. That sets up a one to one mapping of the real numbers to the set of natural numbers, making them a denumerable set.

Explore the detailed proofs and fascinating consequences of $|\chi_0| = |2^{\chi_0}|$ in https://arxiv.org/abs/1002.4433

Addressing mathematical inconsistency: Cantor and Gödel refuted by J. A. Perez.