Infinity and Pythagorean theorem

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Introducing infinity into the Pythagorean theorem provides the Pythagorean theorem even for triangles that are not right triangles.

First, the Pythagorean theorem holds for the three sides of a right triangle.

\[ a^2 + b^2 = c^2 \]

Second, As shown in the figure, the positions of the intersections of the side a and the side b are moved sufficiently large upward while maintaining the lengths of c and d. Due to the relationship of this paper, keep it at a fixed length.

By using the relation shown from the first figure,

\[ k^2 - i^2 = (f + g)^2 - (g - h)^2 \]

\[ l^2 - g^2 = (i + j)^2 - i^2 \]

\[ k^2 - (i + j)^2 = (c - e)^2 \]

Then \( (i+j) = \infty \rightarrow h = 0 \).

The following relationship is obtained from the above three equations.

\[ (f + g)^2 = l^2 + (c - e)^2 \] \hspace{1cm} (1)

From the above equation, by introducing \( \infty \) into a right triangle, the Pythagorean theorem holds for triangles that are not strictly right triangles.

**[Proof]**

Then introduce \( \infty \) including imaginary numbers and compare.

First, \( \pm \infty \) is constant at any observation point (position).

If a set of real numbers is R, then,

\[ R \times (\pm \infty) = (\pm \infty) \]

\[ R + (\pm \infty) = (\pm \infty) \]

\[ (-1) \times (\pm \infty) \neq \mp \infty \]

On the other hand, when \( x (\in R) \) is taken on a number line, the absolute value X becomes larger toward \( \pm \infty \) as the absolute value X is expanded.

Similarly, as the size decreases, the absolute value X decreases toward 0. Furthermore, \( x (-1) \) represents the reversal of the direction of the axis.

\[ \frac{1}{\pm \infty} = (-1) \cdot (\pm \infty) = i \]

\[ (\pm \infty) \cdot i - 1 = 0 \]

\( (-1) \cdot (\pm \infty) = \frac{1}{\pm \infty} \)

\[ i^2 = (\pm \infty)^2 \rightarrow i = \pm (\pm \infty) \]

\[ : i = (\pm \infty) = (-1)(\pm \infty) = \frac{1}{\pm \infty} \cdot (i : \pm (\pm \infty)) \]

Next,

\[ \pi = 2 + 2 \arctan \left( \frac{1}{\tan \frac{1}{\pi}} \right) \]

\[ \pi = 2 + 2 \arctan \left( \frac{1}{\tan \frac{1}{\pi}} \right) = \pi + 2 \arctan \left( \frac{1}{\pm \infty} \right) \]

\[ \arctan \left( \frac{1}{\pm \infty} \right) = \arctan(1) = 0 \]

\[ : \tan 0 = \frac{1}{\pm \infty} = (-1)(\pm \infty) = i \]

From the above figure,

\[ (i)^2 + (-i)^2 = 0 \hspace{1cm} (i : 0 = \text{direction}) \] \hspace{1cm} (2)

\( : i + j = 0, e = i, c = 0, e - c = -i, f = g = h = 0 \hspace{1cm} (0 : \text{direction}) \)

Therefore, it shows that Equation (1) holds in a special triangle in geometry that introduces \( \infty \) and an imaginary number i to the Pythagorean theorem.