A Re-interpretation of the Equation of General Relativity

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Abstract. The basic equation of general relativity is re-interpreted such that, at every spacetime point, the matter-energy tensor is *defined* by the relevant curvature tensor. It is shown that the equation of a geodesic, which is also the equation of motion, then becomes the basic equation governing spacetime dynamics. Implications of the proposed re-interpretation are developed for an elementary – or indivisible – spacetime entity. The properties of mass, spin and charge are thereby seen to have a natural interpretation. A simple experiment is suggested which can corroborate or refute the proposed interpretation, and other implications of the re-interpretation are briefly discussed.

Keywords. Spacetime; spacetime curvature; spacetime dynamics; general relativity.

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1. Introduction

Einstein's foundational equation of the general theory of relativity

$$R_{\mu\nu} - ½Rg_{\mu\nu} = kT_{\mu\nu} \qquad (1)$$

can be interpreted in two fundamentally different ways. The standard interpretation is that the equation describes the way in which matter-energy distribution in spacetime *generates* curvature in spacetime, the effects of which are observable in the form of gravitation. In the words of Gerard 't Hooft [1]:

We have a law of gravity if we have some prescription to pin down the values of the curvature tensor $R^{\mu}{}_{\sigma\alpha\beta}$ near a given matter distribution in space and time.

John Wheeler, in his inimitable style, explained this concept as follows [2]:

Spacetime tells matter how to move; matter tells spacetime how to curve." In other words, a bit of matter (or mass, or energy) moves in accordance with the dictates of the curved spacetime where it is located … At the same time, that bit of mass or energy is itself contributing to the curvature of spacetime everywhere.

Of course this standard interpretation of the general theory of relativity originated with Einstein himself [3]. The new theory had, at first, necessarily to be validated against Newton's law of gravitation. Also, the quantum nature of physical phenomena had not yet been established on firm theoretical basis. It was therefore natural and inevitable that (1) would initially be interpreted in that manner.

This currently prevalent interpretation of the theory of general relativity does not need any explication. A vast amount of experimental evidence – spanning several orders of magnitude in distances – has validated the theory as a great milestone in science, and many exciting areas of research have thereby opened up.

Nonetheless, an argument is presented here in favour of *a fundamentally broader interpretation* of the equation of general relativity. The proposed re-interpretation is fully "backward compatible" with what is already established beyond doubt – but it also compels us to revisit many open and exciting questions in physics, with the possibility of obtaining fresh and immensely valuable insights.

A simple experiment is also suggested, which can either provide clear evidence in support of the proposed interpretation – or refute it conclusively.

2. Re-interpretation of the equation

We first re-write equation (1) as

$$T_{\mu\nu} = (1/k)[R_{\mu\nu} - ½Rg_{\mu\nu}] \qquad (2)$$

and then we interpret the re-written equation as:

What we measure empirically as the matter-energy tensor $T_{\mu\nu}$ at a point in spacetime is nothing but the curvature of spacetime at that point, suitably scaled.

In effect, we say that the mass of a particle is a property which owes its existence to curvature of spacetime. This view implies a radical change in our understanding of spacetime, matter and energy; but it does not displace any established result, whether theoretical or experimental. The re-interpretation is consistent with all the laws of physics in which the mass of a particle or larger body plays its currently accepted role.

However, there is one context in which the proposed re-interpretation makes a prediction quite different from that of the currently established models. The possibility thus arises of designing an experiment to test this prediction, as has been outlined later in this paper.

"Bulk matter" that we deal with on a daily basis owes its apparent bulk to nuclear, inter-atomic and inter-molecular forces. Among macroscopic objects, these forces are tens of orders of magnitude stronger than gravitational forces. So the thesis that, at the elementary level, matter may only be a manifestation of the curvature of spacetime may well appear counter-intuitive.

It is relevant to point out here the well-known assertion of Indian sages: *Tat tvam asi*, or *That art thou*. The assertion is about the absolute oneness of existence. Today, in Quantum Mechanics, it has been understood that the act of measurement brings "the observer" and "the observed phenomenon" together in a way which is essential and inseparable.

In this connection, a famous and amusing incident is also worth noting. Samuel Johnson, "the great lexicographer" of eighteenth century England, was contemptuous of Bishop Berkeley's thesis about the non-materiality of the objects of experience. In an attempt to "disprove" the Bishop's thesis, Johnson once kicked hard at a fairly big stone. As he fell back from ill-advisedly kicking the immovable object, he exclaimed triumphantly to his friend Boswell, "I refute it *thus*!" [4]

It is indisputable that our everyday experience vouches relentlessly for the "materiality" of all the things in and around us. We are so immersed in the sensible qualities of solids, liquids and gases that it is unwise to say to someone, "None of this has any material existence!" A contemptuous response is inevitable.

But physicists have now looked into the deepest underlying nature of physical reality and discovered many properties which would appear counter-intuitive to non-physicists. For example, elementary particles are now seen as quanta of spacetime fields, and the special Higgs mechanism is needed to explain how some of the quanta acquire mass. The present proposal is an attempt to address that same question.

3. Matter-energy Tensor Revisited

In the original formulation of (1), the matter-energy tensor $T_{\mu\nu}$ includes contributions from matter and from the electromagnetic field. In Einstein's words [3]:

But our investigations of the special theory of relativity have shown that in place of the scalar density of matter we have the tensor of energy per unit volume. In the latter is included not only the tensor of the energy of ponderable matter, but also that of the EM energy. We have seen, indeed, that in a more complete analysis the energy tensor can be regarded as a provisional means of representing matter ... From this point of view it is at present appropriate to introduce a tensor, T_{mn}, of the second rank of as yet unknown structure, which provisionally combines the energy density of EM field and that of ponderable matter; we shall denote this in the following as the 'energy tensor of matter'.

First of all, it should be noted that Einstein includes in T_{mn} specifically two types of energy: that of ponderable matter, and that of the electromagnetic field. No mention is made of nuclear forces, although it can perhaps be assumed that they are clubbed together with the energy of ponderable matter.

Secondly, use of the words "... of as yet unknown structure, which provisionally combines ..." may also be noted. In view of the major advances made in the second half of the previous century, Einstein's use of these words indicates clearly that he was aware of the open questions in physics at the time.

In this context, if we consider quantum field theory (QFT), it is clear that "point particles" – handled by the function $\delta(\mathbf{x}-\mathbf{x}_0)$ – and "internal degrees of freedom" are *not* the types of mathematical entities which can be combined with the energy tensor of matter $T_{\mu\nu}$ in a simple way, while maintaining the general covariance required. Indeed, this is definitely one factor which would create serious difficulties in unifying QFT with general relativity.

In the proposed re-interpretation, we include in the matter tensor $T_{\mu\nu}$ only the contributions from massive fields – including of course the rest energy and kinetic energy of massive particles. The contribution from electromagnetic fields is *explicitly excluded* from this tensor.

When we re-interpret (1) in the form of (2), it is necessary to exclude all massless fields from the tensor $T_{\mu\nu}$. This must be so, because in (2) we are proposing that mass is a manifestation of spacetime curvature. That is, we say that equation (2) is no more than a re-labelling of the underlying physical reality, *mass-energy tensor* being another name given to the curvature tensor on the right. It follows that massless fields, which have no impact on the curvature of spacetime, have no role to play in (2).

Contrast with the current interpretation of $T_{\mu\nu}$ is indeed stark. In the current interpretation of general relativity, $T_{\mu\nu}$ includes contributions from all matter and energy fields, massive and massless; but the tensor represents an entity which is somehow "added to" or "slipped into" spacetime. That entity itself – "matter + energy" – is distinct from spacetime, and therefore equation (1) under the current interpretation *relates two disparate entities*.

On the other hand, equation (2) is only a definition of the tensor on the left, in terms of the tensor on the right, which relates to the geometry of spacetime.

Fortunately, there is a simple experiment which should determine the truth or falsehood of the proposed re-interpretation. Consider the experimental setup outlined in Figure 1.

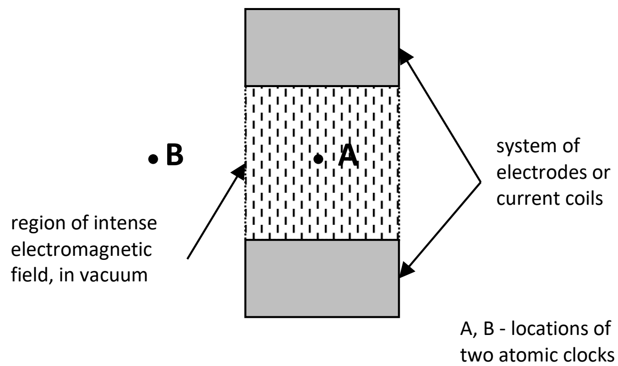

Figure 1 Suggested experiment

Two identical and highly accurate atomic clocks are placed at locations A and B as shown. A is in an evacuated region of intense electromagnetic field, while B is outside. The electromagnetic field is chosen so as not to affect the running speed of the clock at A, and its intensity is made as high as possible. Speeds of the two clocks are measured; the difference in running speed between them, if any, is noted. If necessary, correction is made for the difference in gravitation potential between A and B.

A clear implication of the re-interpretation (2) of the equation of general relativity is that no difference in running speeds should be measured between the clocks at A and B.

At this point, a peculiar property of electromagnetic radiation needs to be noted. While it is true that EM radiation is a form of energy, it is equally true that the energy of EM radiation cannot be harvested until it is absorbed by a massive particle or body. The mediation of a massive particle or body is essential for a photon to manifest itself – that is, to deliver the energy $h\nu$ which it carries. A photon going through a spacetime region free of massive particles leaves no evidence of its passing.

A few examples illustrate the above point: the use of photon energy in photosynthesis, the creation of an electron-hole pair in a solar cell, the generation of a voltage in an antenna, and the heating of a black plate placed in sunlight. Other similar examples can very easily be added to these. The creation of an electron-positron pair is also in the same category.

In view of this property of EM radiation – that is, of its quanta, photons – it follows logically that, in a spacetime region empty of massive particles or bodies, EM radiation would have no impact on curvature. Therefore the experiment outlined above should show no difference between the atomic clocks at A and B.

4. Equation of Motion

In curved spacetime, the equation of motion of an infinitesimal and *isolated* element of mass m at coordinates x_μ, is the equation of the geodesic matching its location and velocity [3]:

$$d^2x_\mu/ds^2 + \Gamma^\mu{}_{\alpha\beta}\,(dx_\alpha/ds)(dx_\beta/ds) = 0 \qquad (3)$$

The symbols used have their usual meaning, and it is assumed that the influence of the mass element m on the spacetime curvature in its immediate neighbourhood is negligible. Under these assumptions, the equation of motion of mass m is independent of the value of m, as was first verified by Galileo.

Here mass m is assumed to be isolated so as to exclude, for the present, its interactions with other masses. In this sense, m is an *elementary spacetime entity*.

Equation (3) remains unchanged under the view implied by (2), but it must be re-interpreted. The infinitesimal element of mass m must be viewed as an infinitesimal volume element dx_ν of spacetime. By (2), the spacetime curvature *of* dx_ν now represents its mass m – since the latter is not any longer a distinct quantity or property which "somehow" attaches itself to the spacetime volume dx_ν. Instead of (1) or (2), under our re-interpretation (3) becomes the central equation of spacetime dynamics.

We consider two cases separately.

Case A. m, the curvature of dx_ν, is a non-negligible quantity

This case represents the reality of most phenomena of immediate interest. Its consistency with established laws of classical and relativistic mechanics has been proved beyond any doubt.

Indeed nothing new needs to be done in this case. The element of spacetime volume dx_ν is to be treated as a particle of mass m moving happily along its geodesic in curved spacetime. Therefore, in this case, the proposed re-interpretation is consistent with all the known physics which follows from (1) & (3).

Case B. m, the curvature of dx_ν, is either zero or a negligible quantity

Since m is a continuous variable, its value can be reduced to zero in a limit operation. The validity of (2) and (3) does not change as m is reduced gradually in a limit operation. Therefore, if we wish *not* to introduce a special case for $m = 0$, then equation (3) applies even when m is negligible or zero.

However, when (3) is applied to the case of m being negligible or zero, the conclusion follows that every element dx_ν of almost or totally flat – "empty" – spacetime moves under the influence of the local spacetime curvature. This conclusion is *not* a part of the conventional interpretation.

Taken together, cases *A* & *B* imply that spacetime *everwhere* – "empty" or otherwise – is under unceasing motion, although it is possible that the detection or measurement of this motion in case *B* presents a huge challenge. It is not wrong to extend the validity of (3) to the case of m being "a heavier entity", provided only that its effect on the spacetime curvature in its neighbourhood is small. In this sense, (3) is seen to represent the *dynamics of spacetime*, with the restriction, for the present, that interactions *within* aggregated mass elements – "bulk matter" – have not yet been considered.

5. Elementary Entity

Consider a region V of spacetime satisfying the following conditions:

(a) The four-dimensional extent of V is symmetrical over the three space dimensions, and also with respect to the time dimension. Under these symmetries, the centre of V can be uniquely defined; its extent is defined as discussed below.

(b) All the geodesics within V remain entirely within V, and satisfy the symmetries listed in (a). All the geodesics outside V remain entirely outside

V. This criterion defines the "boundary" between *V* and the rest of spacetime; no geodesic crosses this boundary in either direction.

(c) We shall assume initially that there is no electrical charge within *V*. However, we shall soon see that a certain attribute of *V* may allow itself to be interpreted naturally as its "charge".

Under these conditions, *V* behaves as an indivisible or elementary entity since, under (b) and the equation of motion (3), no element dx_ν of spacetime ever enters or leaves *V*.

Spacetime curvature falls off on either side of the boundary of *V*, while the mass of *V* is determined by the total spacetime curvature within *V*.

But note also that *V* is *not* a black hole, since it is not "curved enough" to attract other regions of spacetime into it. Thus *V* behaves as elementary or indivisible entity.

Figures 2 and 3 depict this spacetime structure schematically. We shall use the phrase *spacetime potential* in place of the more established phrase *gravitational potential*, since the latter effect can be viewed as but one aspect of the spacetime dynamics which we are exploring.

Geodesics within *V* are closed within the region. This particular fact has huge consequences for the properties of *V*, as we shall now demonstrate by focusing attention on one closed geodesic lying entirely within *V*.

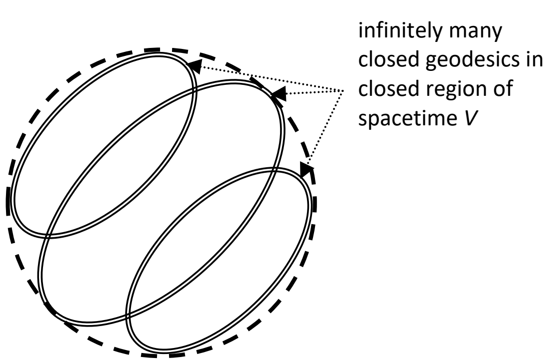

Figure 2 Schematic of an elementary or indivisible entity *V* in spacetime

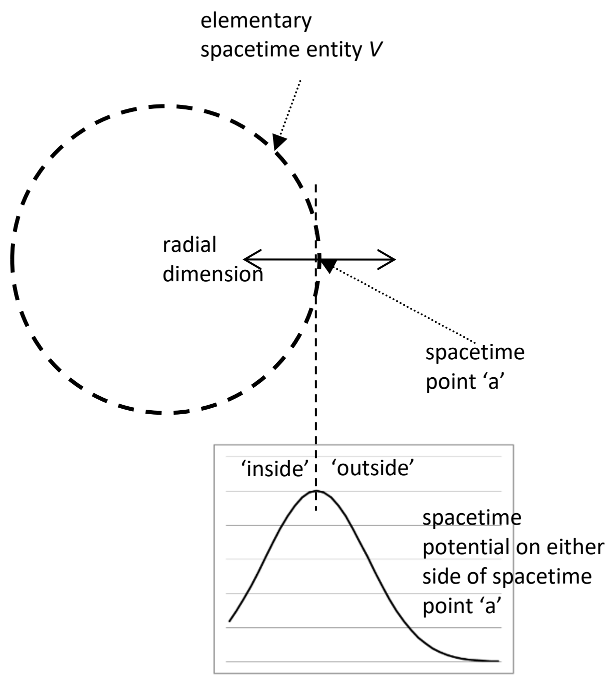

Figure 3 Spacetime potential "at the edge" of *V*

Consider Figure 4, which depicts a closed spacetime geodesic lying within *V*. Elements of spacetime volume dx_ν along the geodesic must move around it, as determined by equation (3). Therefore we see a spacetime "spin" is an essential consequence of equations (2) & (3), and the assumptions (a)-(c) made above.

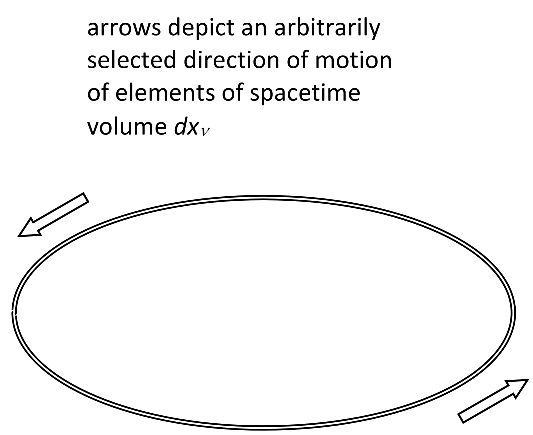

Figure 4 A closed spacetime geodesic within spacetime region *V*

Further, it must also be true that, over "half" of the geodesic, the elements dx_ν are moving backward in time. Therefore a "forward in time" and "backward in time" split is *necessarily* and naturally built into the properties of V.

We see that, under the re-interpretation proposed, the properties of *mass* and *spin* follow naturally from the preceding analysis.

The property of *electric charge* has not yet been explored here, even tentatively. However, we may hypothesize that the right- or left-handedness of the movement of spacetime elements dx_ν within V represent the two types of charge. While much work remains to be done, these three properties do suggest that V has properties of the electron!

5. Discussion

We interpreted the basic equation of general relativity *as a definition of the matter-energy tensor*, rather than the effect on curvature of the matter-energy tensor which is somehow introduced into spacetime. For reasons of logical consistency, it then became necessary to exclude massless fields from the matter-energy tensor. It was then seen that the geodesic equation defines spacetime dynamics.

Analysis of the elementary entity V, as defined above, was made possible because a closed geodesic was considered in the closed off spacetime volume V. Clearly, the analysis would not have been possible under the assumption of "almost flat" spacetime and the use, for example, of Minkowski diagrams. A three-dimensional particle on a world-line in 3+1 representation cannot lead to the conclusion that every elemental volume dx_l within the particle spins, going backward in time over half its trajectory. Under the re-interpretation here, that conclusion follows in a simple and logical way.

It is thus crucial and essential to view physical reality as a dynamic continuum of the three space and one time dimensions. Separation of spacetime into 3+1 dimensions, in a "sufficiently flat" region, may be necessary and useful for performing calculations, but some essential physics is thereby lost. It is hoped that the interpretation proposed herein can lead to further fruitful explorations into the nature of physical reality.

Specifically, in view of the re-interpretation proposed here, it seems plausible that quantum phenomena emerge from specific properties of spacetime dynamics, rather than that spacetime emerges from some deeper quantum phenomena. This issue is central to any attempt to unify the two major modern frameworks of Physics.

In closing, the following statement attributed to American physicist J. W. Gibbs seems quite appropriate [5]:

A mathematician may say anything he pleases, but a physicist must be at least partially sane.

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