Refutation of a formula for systems of Boolean polynomials to parameterized complexity

Abstract: Three formulas defining Boolean polynomial arithmetic are *not* tautologous, to refute the conjecture. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Tomoya Machide, T. (2019). A formula for systems of Boolean polynomial equations and applications to parameterized complexity. arxiv.org/pdf/1907.09686.pdf

Abstract: It is known a method for transforming a system of Boolean polynomial equations to a single Boolean polynomial equation with less variables. In this paper, we improve the method, and give a formula in the Boolean polynomial ring for systems of Boolean polynomial equations. The formula has conjunction and disjunction recursively, and it can be expressed in terms of binary decision trees. As corollaries, we prove parameterized complexity results for systems of Boolean polynomial equations and NP-complete problems.

1 Introduction

The finite field $F_2 = \{0,1\}$ with two elements, which is also called the Galois field GF.. in his honor, plays fundamental roles in mathematics and computer science. It is the smallest finite field with a simple algebraic structure which is determined by a few equations involving the addition "+" and multiplication "·". One of the outstanding facts of F_2 is a structural relation to the two-element Boolean algebra B = {False, True} under the identifications False = 0 and True = 1. That is, for any pair (α,β) of elements,

$$\alpha \wedge \beta = \alpha \cdot \beta, \qquad \alpha \vee \beta = (\alpha + 1) \cdot (\beta + 1) + 1, \qquad \alpha \oplus \beta = \alpha + \beta, \qquad (1.1.1\text{-}.3)$$

where ∧, ∨, and ⊕ stand for the binary operations of conjunction, disjunction, and exclusive disjunction in B, respectively.

LET p, q, r, s: *α , β*, r, s.

(p+q)=(((p+(p=p))&(q+(q=q)))+(s=s)) ;
FTTT **F**TTT **F**TTT **F**TTT (1.1.2.2)

(p@q)=(p+q) ; TTT**F** TTT**F** TTT**F** TTT**F** (1.1.3.2)

2 Review of the Boolean polynomials

… In addition, we have

$$p2 = p, \qquad p(p + 1) = p + p = 0, \tag{2.9.1-.2}$$

(p&(p+(p=p)))=((p+p)=(p@p)) ;

FFFF FFFF FFFF FFFF (2.9.2)

Eqs. 1.1.2.2, 1.1.3.2, and 2.9.2 as rendered are *not* tautologous. This refutes the author's title.