An Anomaly in the Set of Algebraic Real Numbers

by Jim Rock

Abstract: Depending on their representation, the ratio of irrational algebraic numbers to rationals in (***0, 1***) is either ***0.0*** or ***infinite.***

Any decimal in **(*0, 1*)** written in binary notation can be considered an unending decimal with an infinite combination of ***0's*** and ***1's***. The algebraic numbers in **(*0, 1*)** can be partitioned into three different types:

Type 1 – An unending decimal with a finite number of ***1's*** is equal to a terminating decimal **.000110011000… = .000110011** (rational).

Type 2 – An unending decimal that ends in all ***1's*** or ending in a finite pattern ***0's*** and ***1's*** repeated back to back an infinite number of times is a repeating decimal.

Type 3 – An unending decimal that contains an infinite number of ***0's*** and ***1's*** that is not ending in all ***1's*** and not ending in a finite pattern ***0's*** and ***1's*** repeated an infinite number of times is irrational.

Start choosing the algebraic irrational decimals in **(*0, 1*)** one by one.

In the first algebraic irrational we can change all but ***1*** of the ***1's*** into ***0's*** in an infinite number of ways making each into a (type 1) rational.

In the second algebraic irrational we can change all but ***2*** of the ***1's*** into ***0's*** in an infinite number of ways making each into a (type 1) rational.

In the third algebraic irrational we can change all but ***3*** of the ***1's*** into ***0's*** in an infinite number of ways making each into a (type 1) rational.

…

In the n-th algebraic irrational we can change all but ***n*** of the ***1's*** into ***0's*** in an infinite number of ways making each into a (type 1) rational.

This is an unending process. It can be done for every algebraic irrational. In **(*0, 1*)** for every algebraic irrational number there are an infinite number of rational numbers. The ratio of algebraic irrationals to rationals is ***0.0***

Let $\mathbf{rt_n}(a/b)$ be the **n_th** root of ***a/b.*** For every rational fraction ***a/b*** in **(*0, 1*)** there are an endless number of $\mathbf{rt_n}(a/b)$**.** The ratio of algebraic irrationals to rationals is ***infinte.***

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https://arxiv.org/abs/1002.4433

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