The oscillator model, excited states, and electron-photon interactions

Jean Louis Van Belle, *Drs, MAEc, BAEc, BPhil*

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Abstract

This paper explores the implications of the electron oscillator model in regard to the concept of an excited state and the way we think about electron-photon interactions (electron-photon scattering). We also offer some reflections if we can apply the model to a nucleon (a proton or a neutron).

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Introduction

Our oscillator model of an electron[1] combines three equations:

(i) $a \cdot \omega = c$ (radius times angular velocity equals tangential velocity)
(ii) $E = m \cdot c^2$ (Einstein's mass-energy equivalence relation);
(iii) $E = \hbar \cdot \omega$ (the Planck-Einstein relation).

These relations give us the Compton *radius* of an electron:

$$a = \frac{c}{\omega} = \frac{c \cdot \hbar}{\mathrm{m} \cdot c^2} = \frac{\hbar}{\mathrm{m} \cdot c} = \frac{\lambda_C}{2\pi} \approx 386 \times 10^{-15}\ \mathrm{m}$$

We associate this oscillator model with Erwin Schrödinger's trivial solution for Dirac's wave equation for free electrons, for which Schrödinger coined the term *Zitterbewegung*. Dirac summarized it as follows:

> "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

The *Zitterbewegung* concept of an electron combines the idea of a pointlike charge and its motion. As such, it explains the difference between the classical (Thomson) radius of an electron and its Compton radius. Dirac's reference to the 'law of scattering of light by an electron' may, effectively, be a general reference and, therefore, include both Compton as well as Thomson scattering.

Compton scattering involves electron-photon *interference*: a high-energy photon (the light is X- or gamma-rays) will hit an electron and its energy is briefly absorbed before the electron comes back to its equilibrium situation by emitting another photon. The wavelength of the emitted photon will be longer. The photon has, therefore, less energy, and the difference in the energy of the incoming and the outgoing photon gives the electron some linear momentum. Because of the interference effect, Compton scattering is referred to as inelastic.

[1] Jean Louis Van Belle, *The Electron as a Harmonic Electromagnetic Oscillator*, 31 May 2019.

In contrast, low-energy photons scatter elastically. Elastic scattering experiments yield a much smaller effective radius of the electron: the so-called classical electron radius, which is also known as the Thomson or Lorentz radius, and it is equal to $r_e = \alpha \cdot a \approx a/137 \approx 2.818 \times 10^{-15}$ m. We associate this radius with the pointlike charge, while the Compton radius is the effective radius of its local oscillatory motion.

From the above, it is clear that when we say 'pointlike', we do *not* refer to a mathematical point: 10^{-15} m is the *femto*meter scale. That's the order of magnitude of the size of a proton as measured in electron-proton scattering experiments, which is a bit less than 1 fm. Hence, pointlike is small, but not zero.

Form factors and conservation laws

We mentioned that the size of a proton is measured in electron-proton scattering experiments. The reader the proton scatters particles (electrons) that are much *bigger* than itself: the *Thomson* radius of an electron (read: the charge inside) is more than three times *larger* than the proton.[2]

Let us consider the scales involved in electron-photon scattering. Compton's original experiment (1923) involved X-rays, which typically have wavelengths between 0.01 and 10 *nano*meter (10^{-9} m) or gamma-ray photon will have a wavelength, which corresponds to energies between 100 eV and 100 keV. Hence, the wavelength is of an entirely different order of magnitude but the photon *energies* approach those of the electron (511 keV). So what happens here?

A photon packs energy, but it also packs one unit of physical action: Planck's quantum of action h.[3] Hence, as the electron absorbs the photon, it will, for a very brief time, pack *two* units of h. As such, it resembles the *second* Bohr orbital of an electron, which also packs two units of h. What about its angular momentum? Also two units of $\hbar$? No. Here we need to remind ourselves that an electron packs one unit of physical action (h) but only half a unit of angular momentum: $S = h$ but $L = \hbar/2$. The conservation of angular momentum implies our electron should turn into a spin-3/2 particle for a brief moment. In other words, our theory implies its excited state has spin-3/2. Does that make any sense? Perhaps. Perhaps not. We'll discuss this in the next sections.

Let us first think about that 1/2 factor. It is important enough to warrant a small digression. Table 1 summarizes the key formulas for a spin-only electron (a free electron) and an orbital electron respectively. The reader should note we introduce a *form factor* that is equal to 1/2 in the formula for the moment of inertia (I). We used to motivate this by pointing out that our *Zitterbewegung* electron is, effectively, a perpetual current and, hence, it creates a magnetic field, which explains its magnetic moment. The magnetic flux through the ring carries energy and we can, therefore, assume that the electron energy is *not* confined to the current ring. However, the attentive reader will note that the flux *near* the current will be larger than at the center of the ring. This challenges the legitimacy of the 1/2 form factor.

We acknowledge this. At the same time, there are so many other factors at play. We may wonder, for example, why the *Zitterbewegung* orbit would be circular: elliptical orbitals may also be allowed. In fact, the pointlike charge may perhaps pass the center while looping around in some more complicated

[2] See: https://en.wikipedia.org/wiki/Proton_radius_puzzle. The upper and lower estimates of the proton radius are around 0.84 and 0.9 fm respectively. Hence, the ratio of the Thomson radius and the proton radius may be anything between 3.131 and 3.355. It must be a coincidence this ratio is some value around π.

[3] Jean Louis Van Belle, *A Classical Quantum Theory of Light*, 13 June 2019.

orbital (think of the nice shapes of Schrödinger's electron orbitals). We will not digress any further on this, but it is an interesting hypothesis.

Table 1: Intrinsic spin versus orbital angular momentum

Spin-only electron (*Zitterbewegung*)	**Orbital electron (Bohr orbitals)**
$\mathrm{S} = \mathrm{h}$	$\mathrm{S}_n = n\mathrm{h}$ for $n = 1, 2, \ldots$
$\mathrm{E} = \mathrm{m}c^2$	$\mathrm{E}_n = -\frac{1}{2}\frac{\alpha^2}{n^2}\mathrm{m}c^2 = -\frac{1}{n^2}\mathrm{E}_R$
$r = r_\mathrm{C} = \frac{\hbar}{\mathrm{m}c}$	$r_n = n^2 r_\mathrm{B} = \frac{n^2 r_\mathrm{C}}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$
$v = c$	$v_n = \frac{1}{n}\alpha c$
$\omega = \frac{v}{r} = c \cdot \frac{\mathrm{m}c}{\hbar} = \frac{\mathrm{E}}{\hbar}$	$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \frac{\frac{1}{n^2}\alpha^2\mathrm{m}c^2}{n\hbar}$
$\mathrm{L} = I \cdot \omega = \frac{1}{2} \cdot \mathrm{m} \cdot a^2 \cdot \omega = \frac{\mathrm{m}}{2} \cdot \frac{\hbar^2}{\mathrm{m}^2 c^2}\frac{\mathrm{E}}{\hbar} = \frac{\hbar}{2}$	$\mathrm{L}_n = I \cdot \omega_n = n\hbar$
$\mu = \mathrm{I} \cdot \pi {r_\mathrm{C}}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$	$\mu_n = \mathrm{I} \cdot \pi {r_n}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}n\hbar$
$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 2$	$\mathrm{g}_n = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 1$

The excited states of an electron: a naïve model

Let us come back to the idea of excited electron states. When the electron absorbs a photon, we would think that – besides briefly combining the energies of *two* particles (the photon and the electron) – the excited electron will now also pack two units of *h*. How would that work? Can we apply the $\mathrm{S}_n = n \cdot \mathrm{h}$ formula (n = 1, 2, 3, …) to the spin-only electron?

Maybe. Maybe not. Let us try. We no longer have a unique energy and, therefore, we no longer have a unique frequency. Different frequencies imply different cycle times $\mathrm{T_n} = \lambda_n/v_n$. Hence, we have different radii $a_n = \lambda_n/2\pi$ and different tangential velocities v_n. It is just like Bohr orbitals, right? No. If our charge has no rest mass, then its tangential velocity should remain what we assumed it is: the speed of light. Hence, we have a different set of calculations here. Let us try an intuitive approach.

If our equilibrium state – the non-excited state – is written and defined by $\mathrm{E_1 \cdot T_1 = h}$, then our $\mathrm{S}_n = n \cdot \mathrm{h}$ formula implies that $\mathrm{E_2 \cdot T_2 = 2 \cdot h}$, $\mathrm{E_3 \cdot T_3 = 3 \cdot h}$, …, or – more generally - $\mathrm{E}_n \cdot \mathrm{T}_n = n \cdot \mathrm{h}$. If we take the ratio, then we get:

$$\frac{\mathrm{E}_n \cdot \mathrm{T}_n}{\mathrm{E}_1 \cdot \mathrm{T}_1} = n$$

Now, the cycle time is equal to the distance over the loop divided by the velocity: $\mathrm{T}_n = \lambda_n/v_n$. We can, therefore, write the $\mathrm{E}_n\mathrm{T}_n/\mathrm{E}_1\mathrm{T}_1$ ratio as:

$$\frac{E_n \cdot \lambda_n / v_n}{E_1 \cdot \lambda_1 / v_n} = \frac{E_n}{E_1} \cdot \frac{\lambda_n}{\lambda_1} \cdot \frac{v_1}{v_n} = n$$

If v_1 and v_n have to be equal, and equal to the speed of light ($v_1 = v_n = c$), then this might work if $E_n = n^2 \cdot E_1$ and if $\lambda_n = \lambda_1/n$:

$$\frac{E_n}{E_1} \cdot \frac{\lambda_n}{\lambda_1} \cdot \frac{v_1}{v_n} = \frac{n^2 E_1}{E_1} \cdot \frac{\lambda_1}{n \cdot \lambda_1} \cdot \frac{c}{c} = n$$

A *shorter* loop means a *higher* frequency. We can calculate the frequency as $f_n = 1/T_n = v_n/\lambda_n = c/\lambda_n = n \cdot f_1$ or, writing it as an *angular* frequency, $\omega_n = n \cdot \omega_1$. We invite the reader to further cross-check the formulas (Table 2) but, for the time being, they seem to make sense, don't they? Note that the $E_n = n^2 \cdot E_1$ and $\omega_n = n \cdot \omega_1$ tell us that the energy in the oscillation is proportional to the *square* of its frequency, so that sounds perfectly reasonable.

Table 2: The excited states of an electron?

Excited states of a free electron?
$S_n = n\mathrm{h}$ for $n = 1, 2, \ldots$
$E_n = n^2 E_1 = n^2 m_e c^2$
$a_n = \frac{\lambda_n}{2\pi} = \frac{a_1}{n} = \frac{1}{n} \frac{\hbar}{m_e c}$
$v_n = c$
$\omega_n = \frac{c}{a_n} = n \frac{m_e c^2}{\hbar} = n \frac{E_1}{\hbar} = n\omega_1$

These formulas may remind the reader of the textbook explanation of the black-body radiation problem, where energy states were defined as $E_1 = h \cdot f_1$, $E_2 = 2 \cdot h \cdot f_1 = h \cdot f_2$, $E_3 = 3 \cdot h \cdot f_1 = h \cdot f_3$,..., $E_n = n \cdot h \cdot f_1 = h \cdot f_n$. These energy states were all separated by the same amount of energy: $E_n - E_{n-1} = \hbar \cdot \omega_1 = h \cdot f_1 = E_1$. However, it is not the same problem: we have a *square* of *n* in the $E_n = n^2 \cdot E_1 = n^2 \cdot h \cdot f_1$ formula. The square is there because of the velocity factor: $v_n = c$, *always*. The energy *difference* between two orbitals – or two *excitation states*, we should say – can now be calculated as:

$$\Delta E = E_n - E_{n-1} = n^2 \cdot E_1 - (n-1)^2 \cdot E_1 = [n^2 - (n-1)^2] \cdot E_1 = (2n - 1) \cdot E_1$$

ΔE is no longer constant: it is now a linear function of *n*, as shown in Table 3. Also note we get yet another variant of the Ritz combination principle here – but for excited states of the electron instead of electron orbitals.

Table 3: Energy differences: $\Delta E = (2n - 1) \cdot E_1$

n	1	2	3	4	5	…
$E_n = n^2 \cdot E_1$	E_1	$4 \cdot E_1$	$9 \cdot E_1$	$16 \cdot E_1$	$25 \cdot E_1$	
$\Delta E = E_n - E_{n-1} = (2n - 1) \cdot E_1$	E_1	$3 \cdot E_1$	$5 \cdot E_1$	$7 \cdot E_1$	$9 \cdot E_1$	…
$E_3 - E_1$			$4 \cdot E_1$			
$E_4 - E_1$				$15 \cdot E_1$		
$E_5 - E_1$					$24 \cdot E_1$	
…			…			
$E_n - E_1$						$n^2 \cdot E_1 - E_1 = (n^2 - 1) \cdot E_1$
$E_n - E_m$						$n^2 \cdot E_1 - m^2 \cdot E_1 = (n^2 - m^2) \cdot E_1$

This looks quite neat because it establishes a parallel between the emission and/or absorption of a photon as a result of an electron jumping from one electron orbital to another and the emission and/or absorption of a photon as a result of an electron going from one state to another. Let us do an example. The energy difference between the second and first excitation state is equal to:

$$E_2 - E_1 = 3 \cdot E_1 \approx 3 \times 511 \text{ keV} \approx 1.5 \text{ MeV}$$

It is obvious that our model of an excited state doesn't make sense: to go from one state to another, the electron would have to emit or absorb a *multiple* of its own energy ! We are back to square one !

Can we learn anything out of the failure of this naïve model? Probably not. You may think we can do something with this to produce the kind of *transient* particles that are produced in electron-positron colliders but we should probably not entertain such thoughts: electrons and positrons are happy to collide and produce all kinds of stuff, but electrons don't collide with electrons. There is no such thing as a high-energy electron-electron collider. The model we explored here is just what it is: a nice try but non-sensical. We should try something else.

Non-stable particles and the Planck-Einstein relation

We are back to the *hypothesis* we advanced in our previous paper[4]:

Stable particles respect the $E = h \cdot f = \hbar \cdot \omega$ relation – the Planck-Einstein relation – and they do so *exactly*. That's why they are stable.

For non-stable particles – transients – that relation is slightly off, and so they die by falling apart in more stable configurations, until we're left with stable stuff only.

As for resonances – energy blobs with a lifetime of the order of 10^{-22} or 10^{-23}, they are just that: some excited state of a stable or a non-stable particle. Full stop. No magic needed.

Of course, this means angular momentum may not be conserved. At the very least, an incoming photon might change its *direction*.

[4] See: Jean Louis Van Belle, *Smoking Gun Physics*, 21 July 2019.

In other words, the idea that the absorption of a photon briefly turns out electron into a spin-3/2 particle or – equally likely – flips its spin ($\hbar/2 - \hbar = -\hbar/2$) is not plausible. The electron remains what it is, but the incoming photon just disturbs its elementary cycle, and some force in Nature is going to restore that.

What force? Physicists refer to it as the weak force but perhaps we shouldn't think of it as a force.[5] Perhaps it's just time doing its work.[6]

So what's going on, then, when an electron emits or absorbs a photon: the electromagnetic energy of the photon and the electromagnetic energy of the electron just mix and mingle. The end result of the mixing and mingling is the emission of another photon (with the same or lower energy – depending on the energy of the incoming photon) and an electron that's in a different state of motion. That's all. Nothing more. Nothing less.

To conclude this rather short section, we'd like to say something about the neutron. A *free* neutron is *not* stable but its mean lifetime is quite long as compared to the micro- or nano-seconds of other particles: free neutrons have a mean lifetime of about 881.5 ± 1.5 seconds, so that's about 14 minutes and 41.5 seconds (the concept of the half-life of this process (611 ± 1 s) is somewhat different but the order of magnitude is the same). Why would it be stable inside a nucleus? We think it's the Planck-Einstein relation: two protons, two neutrons and two electrons – a helium atom, in other words – are stable because all of the angular momenta in the *oscillation* add up to (some multiple of) Planck's (reduced) quantum of action. The angular momentum of a neutron in free space does not, so it has to fall apart in a (stable) proton and a (stable) electron – and then a neutrino which carries the remainder of the energy. Let's jot it down:

$$\mathrm{n}^0 \rightarrow \mathrm{p}^+ + \mathrm{e}^- + \bar{\nu}_\mathrm{e}^0$$

Let's think about energy first. The neutron's energy is about 939,565,420 eV. The proton energy is about 938,272,088 eV. The difference is 1,293,332 eV. That's almost 1.3 MeV.[7] The electron energy gives us close to 0.511 MeV of that difference – so that's only 40% – but its *kinetic* energy can make up for a lot of the remainder! We then have the neutrino to provide the change—the Euro *cents*, so to speak.

Let's say something about neutrinos here. They are neutral, so what's an *anti*-neutrino? Well… The specialists in the matter say they have no idea and that a neutrino and an anti-neutrino might well be one and the same thing.[8] Hence, we might as well write ν_e. No mystery there—not for me, at least.

The equation above makes it quite tempting to think of a neutron as a proton with an added electron. That works out mass- or energy-wise, as evidenced by the energy numbers above.

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[5] See: Jean Louis Van Belle, *Is the Weak Force a Force?*, 19 July 2019

[6] Time doing its work? Time doesn't do work, does it? A force – acting on a charge over some distance – does work. That's exactly the point I am trying to make. There's no force in play.

[7] CODATA data gives a standard error in the measurements that is equal to 0.46 eV. Hence, the measurements are pretty precise.

[8] See the various articles on neutrinos on Fermi National Accelerator Laboratory (FNAL), such as, for example, this one: https://neutrinos.fnal.gov/mysteries/majorana-or-dirac/. The common explanation is that neutrinos and anti-neutrinos have opposite spin but that's nonsensical: we can very well imagine one and the same particle with two spin numbers.

You will wonder: do I really believe a neutron is the combination of a proton and an electron? My answer to you would be the same question: do *you* think it's possible? Think for yourself ! 😊 It would be nice as an alternative model, wouldn't it? 😊

The Compton radius of a proton

The whole discussion triggers an interesting question: can we build an oscillator model for some other (stable) particle – say, a proton – with our three equations? Let us have a look at them once again:

(i) $a \cdot \omega = c$ (radius times angular velocity equals tangential velocity)
(ii) $E = m \cdot c^2$ (Einstein's mass-energy equivalence relation);
(iii) $E = \hbar \cdot \omega$ (the Planck-Einstein relation).

The Planck-Einstein relation associates a frequency with an energy. Einstein's mass-energy equivalence relation tells us mass and energy are proportional, so we don't have another independent variable there. From a math point of view, this equation doesn't give us anything extra.[9] In short, we have two equation and two variables, and we can reduce this to:

$$a = c/\omega = c \cdot \hbar / E$$

This relation doesn't tell us why the electron has the energy it has. It could be heavier or lighter: we would just find a different Compton radius. Hence, it is tempting to see what other stable particles are around, and see if we can use the same calculation. The Universe doesn't have much stable particles. Even the neutron decays outside of the nucleus. But we do have the proton. A priori, we should *not* expect a sensible result. Why not? Protons stick together – with other protons as well with *neutrons* – inside of the nucleus of an atom and, therefore, some other force – other than the electromagnetic, that is – must be involved: we refer to it as the strong or *nuclear* force. Hence, we would not expect to be able to explain its mass – or its energy – by a pointlike electric charge whizzing around at the speed of light.

It is, therefore, rather remarkable we get a Compton radius for the proton that has got the order of magnitude right. Indeed, if we try the mass of a proton (or a neutron—almost the same) in the $a = \hbar/mc$ formula, we get a radius that's about 1/4 of what's measured in the mentioned proton radius measuring experiments:

$$a_p = \frac{\hbar}{m_p \cdot c} = \frac{\hbar}{E_p/c} = \frac{(6.582 \times 10^{-16}\ \text{eV} \cdot s) \cdot (3 \times 10^{8}\ m/s)}{938 \times 10^{6}\ \text{eV}} \approx 0.21 \times 10^{-15}\ \text{m}$$

The 1/4 factor cannot be explained by the fact our electrons have a size that's at least three times larger than the size of what's being measured. In fact, that's something physicists need to explain to me: how do you get a proton radius of less than 1 fm out of experiments that involve firing particles whose *hard-core* size is equal to 2.818 fm? I must assume the physicists have done their arithmetic – hopefully *not*

[9] That's from a math point of view. The $E = mc^2$ equation does give us the physics of the model!

assuming an electron has no dimension whatsoever[10] - and that they're correct but… Still… It's the kind of explanation I'd like to see in a physics textbook.

Three-body problems, oscillators and symmetries

The ternary structure of the strong force is a bit daunting. We know we don't have an analytical solution for the three-body problem, so how can we hope to make sense of the strong force?

We should make two remarks here. First, there is a very special case of the three-body problem that is referred to as the *elastic* 3-body problem. I'll refer you to an animated *gif*-file – it's one of those animations that is worth a zillion words[11] – that shows starting conditions for the gravitational 3-body problem usually result in chaos. In contrast, there is no such problem (no chaos) for an elastic three-body problem. So we may want to think along those lines.

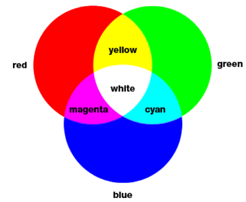
Three bodies? What about the gluons? We think gluons don't exist: all those experiments only yield 'signals' or 'resonances' that are consistent with the theoretical properties of ephemeral transients. We don't need gluons: there is no *conceptual* difference between thinking of a red, blue or green quark and its anti-quark (an anti-red, anti-blue or anti-green quark with opposite *electric* charge) or – a bit simpler – to think of some *parton* with three possible colors and four possible charges. We prefer the parton approach. Why? What's in a name? We just think the concept of some parton that comes in 12 possible varieties (three colors and four charges) separates stuff better.

So we want to make particles of partons. We need to introduce some rules, of course. One of them is that the charges have to add up to the elementary charge (+1 or −1) or – for neutral particles – have to equal zero. That's where the *anti*-color in the quark-gluon model comes in, but we don't want to think in terms of anti-colors. The electric charge rule will do. What about our white-color rule? We can drop that for the time being. If we allow red to combine with itself and with blue and green, we get a matrix. To be precise, the strong force may be different for red and red, red and green, and red and blue, so we can put some coefficients in.

	red	**green**	**blue**
red	$S_{\text{red-red}}$	$S_{\text{red-green}} = S_{\text{green-red}}$	$S_{\text{red-blue}} = S_{\text{blue-red}}$
green	$S_{\text{green-red}} = S_{\text{red-green}}$	$S_{\text{green-green}}$	$S_{\text{green-blue}} = S_{\text{blue-green}}$
blue	$S_{\text{blue-red}} = S_{\text{red-blue}}$	$S_{\text{blue-green}} = S_{\text{green-blue}}$	$S_{\text{blue-blue}}$

We have nine coefficients but only six of them will be independent. This is actually where the color mixing picture comes to mind: red and blue makes purple (or, to be precise, magenta), red and green makes yellow, and green and blue makes blue-green (which is also referred to as cyan). So we have three primary colors and three mixed colors.

[10] After having de-constructed some of Feynman's arguments (see: Jean Louis Van Belle, *The Double Life of −1*, 30 October 2018), I don't take anything for granted anymore (including Nobel Prize physics). I advise you to do the same: be critical.
[11] See: https://commons.wikimedia.org/wiki/File:3bodyproblem.gif#/media/File:3bodyproblem.gif



If you know anything about QCD, the matrix may make you think of the Cabibbo–Kobayashi–Maskawa matrix, but it's got nothing to do with it: that matrix gives you the probability (or amplitude) for the *flavor* (u, d, c, s, t, b) to change into another. As for now, we don't think we need quark *flavors* to explain transient particles. We have enough degrees of freedom here.

We should probably remind ourselves of the properties of a symmetric matrix here: An n-by-n symmetric matrix will have n eigenvalues, and we can then find a set of n eigenvectors – one for each eigenvalue – that are mutually orthogonal. The matrix here is a 3-by-3 matrix: something inside of me tells me this should explain the three *generations* of matter in the Standard Model.

The electric charge rule – the electric charge has to add up to +1, 0 or −1 – should then explain the rest. The concepts of quarks, gluons or flavors sounds a bit like the *aether* theory. The philosophical concept of a colorless, flavorless and zero-charge parton – onto which we can then load the various properties we need to explain reality – may work just as well.

What about Yukawa's $e^{-r/a}$ factor in the force and/or potential function? We can add it. In fact, the easiest functional form for the six color coefficients would be one with an $e^{-r/a}$ factor in which the *range* parameter a depends on the color charges.

We could further delve into this, but we don't not want to go too far off-track. When everything is said and done, the objective of this paper was only to do so thinking about excited states. We think we covered that subject for the time being.

Jean Louis Van Belle, 24 July 2019