Refutation of the orthomodular law

Abstract: We evaluate the orthomodular law x≤y implies y=x ∨ (y ∧ x′), with ′ as negation, which is *no*t tautologous. This forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Chajda, I.; Länger, H. (2019). arxiv.org/pdf/1907.10539.pd
How to introduce the connective implication in orthomodular posets.

Abstract: Since orthomodular posets serve as an algebraic axiomatization of the logic of quantum mechanics, it is a natural question how the connective of implication can be defined in this logic. It should be introduced in such a way that it is related with conjunction, i.e. with the partial operation meet, by means of some kind of adjointness. We present here such an implication for which a so-called unsharp residuated poset can be constructed. Then this implication is connected with the operation meet by the so-called unsharp adjointness. We prove that also conversely, under some additional assumptions, such an unsharp residuated poset can be converted into an orthomodular poset and that this assignment is nearly one-to-one.

Orthomodular posets are considered as an algebraic axiomatization of the logic of quantum mechanics … On the other hand, when some algebraic structure is used as an axiomatization of a propositional logic, we must ask for a connective implication … In the present paper we solve the question of finding an implication in orthomodular posets in the way that a certain residuation is possible.

Recall that a *bounded poset with an antitone involution* is an ordered quintuple (P, ≤ , ′, 0, 1) where (P, ≤, 0, 1) is a bounded poset and ′ is a unary operation on P such that the following conditions are satisfied for all x, y ∈ P: x ≤ y implies y′ ≤ x′, (x′)′ = x.

Remark 1.0: The mark ′ is effectively the negation operator ¬.

We say that the elements a, b of P are *orthogonal* to each other if a ≤ b′ (or, equivalently, b ≤ a′). Further recall that an *orthomodular poset* is a bounded poset (P, ≤, ′, 0, 1) with an antitone involution satisfying the following conditions for all x, y ∈ P: x ∨ y is defined provided x ≤ y′, [and]

• x ≤ y implies y = x ∨ (y ∧ x′). (1.1)

LET p, q: x, y.

`~(q<p)>(q=(p+(q&~p)))` ; T**F**TT T**F**TT T**F**TT T**F**TT (1.2)

The last condition is called the *orthomodular law*. Observe that in case $y = 1$ this law implies $x \vee x' = 1$. Since $'$ is an antitone involution this further implies $x \wedge x' = 0$. Thus $'$ is a complementation.

Remark 1.2: Eq. 1.2 as rendered is *not* tautologous, hence refuting the orthomodular law.