Refutation of the Borel base and hull

Abstract: We evaluate in two equations the Borel base and hull as *not* tautologous and contradictory, refuting the conjectures and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊭, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Michalski, M. (2019). Rediscovered theorem of Luzin. arxiv.org/pdf/1907.09305.pdf

Definition 1.1. We say that a σ-ideal I

has a Borel base if (∀A ∈ I)(∃B ∈ B ∩ I(A ⊆ B); (1.1.1)

LET p, q, r, s: A, B, B', I.

(#p<s)&((%q<q)&(s&~(q<p))) ;
FFFF FFFF FFFF FFFF (1.1.2)

has a Borel hull property if for (∀A)(∃B ∈ B)(A ⊆ B and (∀B′ ∈ B)(A ⊆ B′ ⊆ B)(B\B′ ∈ I)). ... (1.2.1)

#p&((%q<q)&(~(q<p)&((#r<q)&(~(q<~(r<p))&(q\(r<s)))))) ;
FFFF FFFF FFFF FFFF (1.2.2)

Eqs. 1.1.2 and 1.2.2 as rendered are not tautologous, refuting the Borel base and hull conjectures.