

Energy-momentum is not defined globally, but locally

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Abstract

Slow precession of the Earth rotation axis, and the Moon-Earth orbital resonance were accompanied during centuries by Newton's and Laplace's explanations. However, in the present paper, the author considers the possibility of additional factor: the small violation of the global energy-momentum conservation. Thus, the energy-momentum concept being not conserved, cannot be regarded as total (global) energy-momentum of system.

The recently experimentally verified Lense-Thirring Effect and the Mercury's perihelion anomalous shift cannot be found in Newton Physics, and the latter demands the global energy-momentum conservation. Thus, the shift violates global energy-momentum conservation. Why? Because the energy-momentum is defined locally, not globally.

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In my definition, the energy-momentum non-locality implies that if we summed up all the local energy-momentum values, we should have got the global energy-momentum value. But it turns out not as constant. Consequently, it is not an energy-momentum. Consequently, the energy-momentum is always local. And if we insist that stars in the galaxy scale be subjected to Newton's Physics, we require the globalization of the energy-momentum. But we are making a mistake. "Dark matter" is the name of the mistake. Latter came out on a global (thus non-local) scale: Dark matter is, yes, non-local.

Well, indeed, consider two closely spaced free bodies in orbit around the Sun. The imaginary axis connecting these bodies will make a year-long change of direction, as these bodies move around the Sun. However, if this axis would pass through a rapidly rotating body (like our Earth), then it would always be directed to the North Star (due to the law of conservation of angular momentum in Newton Physics, which can consider entire Solar System in a big closed box). Conclusion: The precession of the Earth Axis is influenced by a Tidal Gravitational Effect, which can only be described in Einstein's Theory. [1]

I. RECOMMENDED METHOD TO READ THE PAPER

Do not be lazy, read the every single word; and the method to read this paper you get from this lecture in scientific psychology.

To my definition, the Love is God, because Love unites two people. Love Unites. Therefore, True Love is always mutually shared. Non-shared Love from your side can not be called love, but rather the addiction. But then you might ask "the Bible commands to love even your personal enemy, he is not sharing the love with you, is he?" No, the enemy does not share the love with you, but the love to the enemy is in fact the love to God. Because you love your God (mutually), and all His plans then.

Indifference does not originate from God. From God comes only Love. Perfect Love. The Love at first sight: "God commendeth his love toward us, in that, while we were yet sinners, Christ died for us." Romans 5:8. To a Bible researcher: and because Love is defined as mutually shared, the God's love to His enemy means, that Love is in fact God.

What you heard about love at first sight? In the Bible God tells us to love even personal enemies (at first sight). The Greeks divided Love into many kinds. No, there is only one Love, because He is the Undivided Holy Trinity. I like to define things and concepts, because

Reality is based on definitions. The friendship between a guy and a girl is the real Love; Love, but before marriage. You agree? People are longing to the love at first sight: “Little Big Town - Happy People” YouTube.

In my working place I am a scientist with expertises. The current method of science “scientific skepticism” is not true to Love. It is not Love to authors, who submit research papers to journals, at all. The method of Love is in fact the “scientific trust”, and it is always applied to the chosen, prominent authors like Stephen Hawking: all what Stephen says will be published for sure. The beginners in science need to publish under authorship with prominent supervisor, otherwise they will be subjected to severe scientific skepticism.

The unhealthy addiction to scientific skepticism pushes the reader to see a lot of mistakes, inconsistencies, and dubious places in the paper. So now please read text with a strong desire to confirm it: be more like a friend, not an alien referee. If the author says, that he has written a proof, then a polite human considers it as proof. Did not the reader hear about the human factor – wishful thinking? Latter is the cause of papers questioning global warming. Try to love the author like “Good Samaritan” does to a stranger and look not for refutations (if desired, everything can be refuted through over-pushed imagination, trivial trolling, lies and daily discrimination), but look to confirm the manuscript. However, the globally overused method “scientific skepticism” pushes an author to earn tons of Gold and Platinum to convince editors in his sanity: that the newest open access (with astronomical article processing fees) journals are for.

The reader thinks that we must keep the old system and methods even if they are not entirely healthy for authors because we have no alternatives. No. The method of “scientific trust” is this:

1. While reading the manuscript, a referee felt that a place in the article is doubtful.
2. The referee tries to justify his unpleasant feeling with logic.
3. Having substantiated it with logic, the referee looks for the reasons why the author may still be right within the arguments of the paper: so, e.g., the author is not forced to write appeal like “the referee forgot the text at Eq.(2)!”
4. If there are no such reasons for all referee’s desire to serve the author’s interests, then reject the article.

A. To prove you, that I am “genius”

To make you trust my mind, I am giving you now the short proof of Fermat’s Last Theorem.

“Fermat’s Last Theorem is just about numbers, so it seems like we ought to be able to prove it by just talking about numbers,” McLarty said. [3] “I believe that can be done, but it will require many new insights into numbers. It will be very hard.”

The known proof [4] uses several assumptions like the existence of “big cardinal numbers” [3]. If it is necessary to use assumptions, why then we can not take the entire Fermat’s Theorem at face value? Just call Fermat’s Last Theorem a Postulate. Like was done with the 5-th postulate of Euclid. In author’s understanding, a postulate is different from Axiom, because falsification of it does not falsify basic theory; and a postulate is for a long time not proven conjecture, with this a young conjecture cannot be a postulate. Thus, one has the right to Postulate Fermat’s Theorem, because known proof of the latter uses unproven assumptions. On the contrary, the following text has no assumptions and no complexity.

Because is proven by Euler [5], that $a^3 + b^3 = c^3$ does not have solution for integers a, b, c , then $u^3 + v^3 = 1$ does not have solution for rational u, v . Thus,

$$(z^\kappa)^3 + (y^m)^3 = z^{3\kappa} + y^{3m} = 1, \quad (1)$$

where integers $\kappa \geq 1, m \geq 1$, has no solution in rational z, y ; because then z^κ and y^m are both rational.

If Fermat’s Conjecture is right, that

$$q^n + w^n = 1 \quad (2)$$

does not have solution in rational numbers if $n \geq 4$, then as direct consequence is proven that

$$g^{3n} + p^{9n} = 1, \quad (3)$$

where $n = 4, 5, 6, \dots, \infty$ does not have solution in rational numbers. The Fermat’s Conjecture becomes deduced to the proven theory of Eq.(1) with $\kappa = n, m = 3n$.

Therefore, the author suggests to include the following Axiom into Number Theory: suppose we have a situation with two Conjectures A and B, such that if Conjecture A is true, then must be true Conjecture B. Then, if isomorphism can be established between the

conjectures, holds the following: if Conjecture B becomes proven, then Conjecture A must be true. Indeed, the conjectures described by Eq.(2) and Eq.(3) are isomorphic equivalent: $q = g^3$ and $w = p^9$.

You can always find a violation of the Logic of Aristotle in any hazy text. In any. No one found that in my text.

II. ON ENERGY LOCALIZATION PROBLEM

By recalling the basic need to study problems in an inertial coordinate system (tetrad) [recall the demand for an inertial tetrad in the Galilean and Einstein Postulates of Relativity: in a non-inertial tetrad would be made accelerative change to Standard Instruments (meter, clock, amperimeter, etc.), but that comes in conflict with Metrology [15]], we found no problem with the local conservation of the most basic laws of physics. But others have faced major problems (cf. e.g. Refs. [16]).

The vector of rate in the local (ON: orthonormal) tetrad has

$$\frac{d B^{\hat{\nu}}}{d\tau} = e_{\alpha}^{\hat{\nu}} \frac{D B^{\alpha}}{d\tau}. \quad (4)$$

Thus, if $B^{\hat{\nu}}$ conserves in inertial tetrad, then

$$\frac{d B^{\hat{\nu}}}{d\tau} = 0, \quad \frac{D B^{\alpha}}{d\tau} = 0. \quad (5)$$

But because

$$B^{\alpha} = e_{\hat{\nu}}^{\alpha} B^{\hat{\nu}}, \quad (6)$$

then the inertial tetrad is defined by

$$\frac{D e_{\hat{\nu}}^{\alpha}}{d\tau} = \frac{d e_{\hat{\nu}}^{\alpha}}{d\tau} + \Gamma_{\beta\gamma}^{\alpha} e_{\hat{\nu}}^{\beta} u^{\gamma} = 0. \quad (7)$$

Working in the Schwarzschild metric, I have managed to find the following co-moving coordinates which are indeed able to show during one year on the North Star area:

$$\begin{aligned} e_{\mu}^{\hat{t}} &= \left(\frac{4\sqrt{70}}{35}, 0, 0, -\frac{10}{\sqrt{7}} \right), & e_{\mu}^{\hat{r}} &= \left(\frac{2}{\sqrt{35}} \cos(w\tau), -\frac{\sqrt{5}}{2} \sin(w\tau), 0, -\frac{20\sqrt{14}}{7} \cos(w\tau) \right), \\ e_{\mu}^{\hat{\theta}} &= (0, 0, r, 0), & e_{\mu}^{\hat{\phi}} &= \left(\frac{2}{\sqrt{35}} \sin(w\tau), -\frac{\sqrt{5}}{2} \cos(w\tau), 0, -\frac{20\sqrt{14}}{7} \sin(w\tau) \right), \end{aligned} \quad (8)$$

where $w = \sqrt{10}/100$, $M = 1$, $r = 10 = \text{const}$.

That enlightenment solves the Energy Localization problem in General Relativity. Indeed, the known formula

$$T^{\nu\mu}_{;\nu} = 0 \quad (9)$$

in inertial ON tetrad is the needed conservation of energy-momentum

$$T^{\hat{\nu}\hat{\mu}}_{,\hat{\nu}} = 0. \quad (10)$$

But latter means, that in inertial ON tetrad all the Christoffel Symbols are zero

$$\Gamma^{\hat{\alpha}}_{\hat{\nu}\hat{\mu}} = 0. \quad (11)$$

III. DARK MATTER SOLUTION

The talk of the energy-momentum conservation on a galactic scale means the application of it at a global scale. However, such application is a mistake; thus, only the local application uncovers the Dark Matter nature. The tests in CERN and Earth Dark Matter Detectors are local tests, which do not reveal the Dark Matter. Thus, the latter are not real particles, but rather the effect of the wrong scale application of energy-momentum conservation. Please read the paper, how the energy conservation looks to be violated at the global scale: Ref. [17]. Latter reference proposes to modify theories to explain Dark Energy; so, following their agenda, the current paper proposes the most natural and simple modification of Einstein Equations:

A. Virtual Reality

We know that there are many objects of baryonic (physical) matter with energy-momentum tensor \mathbf{T} : stars, planets, etc. Baryonic matter satisfies the Einstein equations, where Dark Matter can be written in. Dark matter is described then by a certain tensor $\mathbf{DM}(t, x, y, z)$ to be determined yet, where the coordinates are given on the map of the Universe. Dark matter is not subject to the energy conditions (weak, strong, and others), because nobody will record the violation: Dark matter does not interact with baryonic matter other than gravitationally. We have to work hard to determine the ten unknown functions $DM^{\mu\nu}$. Just invent for any particular problem a suitable dark matter tensor: we need to match theories with Nature.

Instead of inventing the **DM** of dark matter we can invent the metric tensor **g** and easily calculate from it the Einstein tensor **G**. This way one has $8\pi \mathbf{DM} = \mathbf{G} - 8\pi \mathbf{T}$. The **DM** has zero covariant divergence, $DM_{;\mu}^{\mu\nu} = 0$. Therefore, this tensor is a special case of virtual matter which can have a tensor **VM** with $VM_{;\mu}^{\mu\nu} \neq 0$. This mathematical term **VM** in $\mathbf{G} = 8\pi\mathbf{T} + 8\pi\mathbf{VM}$ is not physical: the virtual matter, being just mathematics, does not correspond to a physical object and, thus, is not subject to energy conditions (this fact could lead to possibilities of interstellar travel using spacetime wormholes or Alcubierre warp-drives); unlike it, the **T** corresponds to stars, planets, etc. Therefore, the virtual matter can act on baryonic matter **T** without being directly observable. The virtual matter is a particular case of virtual terms **VT**. The latter are mathematical modifications of the equations [18], not only of the form $\mathbf{G} + \mathbf{VT} = 8\pi \mathbf{T}$. There is only one demand for virtual terms: they must not cause paradoxes in Nature. Accordingly, dark energy is a particular form of dark matter.

Virtual reality is defined as mathematical inserts in equations. Therefore, a virtual object is not directly detected by measuring instruments. Here there are underground detectors of particles of Dark Matter, but they showed nothing.

B. On energy-momentum of Dark Matter: advice for the future studies

I argue, that the yet used form of Dark Matter as perfect fluid (with non-zero elements on diagonal $\text{diag}(\rho, p, p, p)$) is not the only form of energy-momentum tensor of Dark Matter. Look, the accompanying it mystery, the Dark Energy is also not a perfect fluid, and it has quite strange and unique energy-momentum ($\Lambda \text{diag}(1, 1, 1, 1)$), which has not yet found any correspondence with any configuration of Standard Model of particles.

Moreover, is known, that wormholes, which allow spacetime shortcuts, do use the so called “exotic” matter. Such matter violates the so called “energy conditions”, but in case if it is the energy-momentum of Dark Matter, there are no problems with measuring the negative energy, because Dark Matter does not interact (other than by gravity) with baryonic Standard Instruments, like the calorie-meter (which is made to measure energy).

We can not argue over the form of energy-momentum tensor, so any assumptions about it’s form must be published. And is allowed then, that Dark Matter does not have certain form of energy-momentum tensor: we are free to supply it with any needed form in a

particular problems.

IV. THE USE OF VIRTUAL TERMS

As shown by Laura, Hawking radiation can stop a star collapse, so there are bodies that are larger than the Schwarzschild sphere but smaller than the neutron star [10]. In this note I give another explanation: due to dark matter [11] the ergosphere is the stable surface of a body. In that way, we avoid having (hypothetical) negative energy particles in the Penrose energy extraction process inside the ergosphere, and we are getting rid of the Hawking information loss paradox at the “bottom” of the ergosphere, i.e., at the event horizon. Is expected then, that such a body is the black hole in the middle of the galaxy M87.

A. Is it black hole in the middle of the galaxy M87?

Then for properly chosen Virtual Matter field $\mathbf{VM}(t, x, y, z)$ the gravitational collapse can produce the black body (inside the galaxy M87) not with the event horizon is being black, but with the black ergosphere. If the surface of the body is the ergosphere, we can notice that the speed of any falling matter is the speed of light just on the surface of the body (measured by a stationary observer [8]). I have calculated this speed using Ref. [9]. Therefore, there is an effective red-shift, and the surface of the body must be black. Indeed, by definition of ergosphere surface there is $g_{tt} = 0$, and the metric for stationary observer (he has $d\phi = d\theta = dr = 0$) just at the surface reads $d\tau^2 = g_{tt} dt^2$, which looks exactly like the cause for the infinite red-shift turning black body “black”.

The Event Horizon Telescope Collaboration judges the mass of the black hole by the size of the black spot (they are using word “shadow”) in the sky. But if it is the ergosphere black, not the event horizon (because latter is absent for a body, which surface is ergosphere); then judging by the size of the black spot, the mass of “extremal” black hole will be twice times smaller than reported by Event Horizon Telescope Collaboration, because ergosphere surface is twice larger than the event horizon: $r_E = 2r_h$.

As a consequence, the mass value $m = 6.5 \pm 0.2 \pm 0.7$ billion solar masses, as reported by the event horizon Telescope Collaboration [6], can be divided by a factor of two to produce

the correct mass $M = m/2 = 3.25 \pm 0.1 \pm 0.35$. Therefore,

$$3.25 - 0.1 - 0.35 = 2.8 < M < 3.7 = 3.25 + 0.1 + 0.35.$$

This range perfectly agrees with the previous most recent mass determination [7], which was

$$2.8 < M < 4.4.$$

However, we have noticed that the precision of our instruments has noticeably grown over the years, $3.7 < 4.4$. The author offers this solution to solve the discrepancies between the results of Refs. [7] and [6].

So again, the Event Horizon Telescope Collaboration reports the mass m of the black hole by measuring the proper size ψ of the black spot in the middle of the Galaxy M87 by the formula: $m = \psi$ in meters. However, the author has presented evidence, that the size of the black spot relates to true mass M as $\psi = 2M$ for “extremal” black hole. Because the Telescope sees being black not the event-horizon at $r_h = M$, but the ergosphere at $r_E = 2M$. Therefore, $M = m/2$. So, the author suggests renaming Event Horizon Telescope into “Ergosphere Telescope”.

B. Virtual Term is needed for the solution to Navier-Stokes Millennium Prize problem

We consider a perfect fluid in fixed flat spacetime with Descartes coordinates (x, y, z) . Alternatively, one can transform the coordinates of the curved spacetime to have local Minkowski metric with zero Christoffel Symbols at given point. [12] Such transformation produces the inertial ON tetrad.

The energy-momentum tensor of the perfect fluid [9, 13] reads

$$T^{\nu\mu} = (\rho + p) u^\nu u^\mu + p g^{\nu\mu} \quad (12)$$

with $u^\nu u_\nu = -1$. Then $T^{\nu\mu}_{,\nu} = 0$ means

$$0 = u_\mu T^{\nu\mu}_{,\nu} = \frac{d\rho}{d\tau} + (\rho + p) \Theta, \quad (13)$$

where $\Theta = u^\nu_{,\nu}$ and

$$\frac{d\rho}{d\tau} = \frac{\partial\rho}{\partial x^\nu} u^\nu. \quad (14)$$

Let us calculate the density current [9, 12]

$$J^\mu = -T^{\nu\mu} u_\nu = \rho u^\mu. \quad (15)$$

Then

$$J^\mu_{,\mu} = \frac{d\rho}{d\tau} + \rho \Theta, \quad (16)$$

and so from Eq. (13)

$$J^\mu_{,\mu} = -p \Theta. \quad (17)$$

However, it is known that in flat spacetime the continuity equation for density current $J^\mu_{,\mu} = 0$ holds [9, 13, 14]. Therefore, $p = 0$ which is a violation for the fluid: it is no longer fluid but dust! It explains the Albert Einstein sorrow, that there is no stationary universe model filled with fluid if Dark Energy with its Cosmological Constant is absent. Note, that the case $\Theta = 0$ in Eq. (17) would mean that $\rho = \text{const}$ for any pressure p (cf. Eq. (16)). However, an absolutely rigid object is not allowed because the speed of interactions is finite.

Because the perfect fluid is the case of a Navier-Stokes viscous fluid equations, then Navier-Stokes fluid equations are proven to be inconsistent now: a Virtual Term is necessary.

V. VANISHING OF ACTUAL (PHYSICAL) MATTER INTO VIRTUAL STATE

Because there are singularities in General Relativity, the latter can hardly be regarded (in current state of affairs) as a predictable and conservative theory. Indeed, the vanishing of a body sometimes occurs at a finite curvature of spacetime. The velocity component is given by [9]

$$u^r \equiv \frac{dr}{d\tau} = -\frac{1}{r^2} \sqrt{B}, \quad (18)$$

where $B = E^2 r^4 - (r^2 - 2 M r + Q^2) r^2$.

In “geometrized” units (Q, M, r in meters) let us choose $Q = 1/5$ and $M = 1/2$. Zero initial velocity ($B = 0$ at $r = r_0 = 20$) requires a trajectory with

$$E = \frac{\sqrt{9501}}{100}. \quad (19)$$

Therefore

$$B = -\frac{499}{10000} r^4 + r^3 - \frac{1}{25} r^2, \quad (20)$$

which is negative in $r < r_m = 20/499$. This means a termination of the falling body. My detailed study shows that even photons are being terminated (in the Kerr-Newman spacetime; but various terminations are present also in a Kerr spacetime as well as in naked singularity regimes). Such obvious termination was never reported, cf. e.g. Refs. [20], because the Virtual Terms Theory is needed.

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