**Refutation of bounded homomorphisms and finitely generated fiber products of lattices**



**Abstract:** We evaluate two equations for the standard method for constructing subdirect products, which are *not* tautologous. Hence the conjecture of bounded homomorphisms and finitely generated fiber products of lattices is refuted. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~( y < x) ( x ≤ y), ( x ⊆ y), ( x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Demeo, W.; Mayr, P.; Ruškuc, N. (2019). Bounded homomorphisms and finitely generated fiber products of lattices. arxiv.org/pdf/1907.08046.pdf

**Abstract.** We investigate when fiber products of lattices are finitely generated and obtain a new characterization of bounded lattice homomorphisms onto finitely presented lattices and onto lattices satisfying Whitman's condition. Specifically, for lattice epimorphisms g : A → D, h: B → D, ... we show the following: If g and h are bounded, then their fiber product (pullback) C = {(a, b) ∈ A × B | g(a) = h(b)} is finitely generated. While the converse is not true in general, it does hold when A and B are free. As a consequence we obtain an exponential time algorithm to decide whether a finitely presented lattice or a finitely generated sublattice satisfying Whitman's condition is bounded. This generalizes an unpublished result of Freese and Nation.

We start by recalling a standard method for constructing subdirect products.

Let A, B be algebras with epimorphisms g: A → D and h: B → D onto the same homomorphic image D. Then the subalgebra C := {(a, b) ∈ A × B | g(a) = h(b)} of A×B is called a fiber product (or pullback) of g and h. Clearly C is a subdirect product of A and B. (1.1.1)

LET p, q, r, s, t, u, v, w: A, B, C, D, a, b, g, h.

((v=(p>s))&(w=(q>s)))>(r=(((t&u)<(p&q))>((v&t)=(w&u)))) ;

TTT**F** TTTT TTTT TTTT( 4)
TT**F**T TTTT TTTT TTTT( 3)
TTTT TT**F**T TTTT TTTT( 1)
T**F**TT TTTT TTTT TTTT( 3)
TTTT T**F**TT TTTT TTTT( 1)
**F**TTT TTTT **FFFF** TTTT( 4) (1.1.2)

**Remark 1.1:** If g and h are substituted with the respective expansions, then Eq. 1.1 reads as:

$$C := \{(a, b) \in A \times B \mid (A \to D)(a) = (B \to D)(b)\} \tag{1.2.1}$$

r=(((t&u)<(p&q))>(((p>s)&t)=((q>s)&u))) ;

**FFFF** TTTT **FFFF** TTTT( 3)

**F**TT**F** T**FF**T **FFFF** TTTT( 1) (1.2.2)

Eqs. 1.1.2 and 1.2.2 are *not* tautologous. This refutes the standard method for constructing subdirect products, and hence the conjecture of bounded homomorphisms and finitely generated fiber products of lattices.