Testing the Number of space-time dimensions by the 5.9 years repeated Millikan's oil drop experiments

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Abstract. The basic motive of the five-dimensional Kaluza–Klein theory is the unification of gravity and electromagnetism. A feature of these theories was the relation between the electromagnetic coupling e^2, and gravitational coupling G_N and the radius of the fifth dimension R_c. The radius of the fifth dimension R_c is thus fixed by the elementary electric charge. From the known value of the elementary charge, we find that R_c is of the order of the Planck length. Based on The five-dimensional Kaluza–Klein theory, we show that if the observed harmonic pattern of the laboratory-measured values of G_N is due to some environmental or theoretical errors, these errors must also affect the elementary electric charge e. We calculate the values of fundamental electric charge e predicted by 3+1 and 4+1 dimensional space-time model respectively. We find that in the case of 4+1 the fundamental electric charge e values are oscillated with the 5.9 year LOD oscillation cycle, while in the case of 3+1 space-time dimensions the fundamental electric charge e is constant and perfect fitted to the straight line. Furthermore, we propose that the number of space-time dimensions can be reveal by the 5,9years repeated Millikan's oil drop experiments.

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1. Introduction

Newton's gravitational constant, *G*, has been measured about a dozen times over the last 40 years. Recently, John D. Anderson and coauthors [1] found that the measured *G* values oscillate over time like a sine wave with a period of 5.9 years. They propose that this oscillation of measured G values does not register variation of G itself, but rather the effect of unknown factors on the measurements [46]. C.S.Unnikrishnan [47] provides a possible explanation to the 5.9-year period of G values oscillation by the gravitational link between the

Earth and Jupiter: there is a 5.9 year periodicity in the length of the Earth's and an 11.86 year periodicity in Jupiter's around the Sun.

Klein (2015[2]) suggests that the observed discrepancies between *G* values determined in different experiments may be associated with a differential interpretation of Modified Newtonian Dynamics (MOND) theory applied to the galaxy rotation curves. Recent quantitative analysis (Lorenzo Iorio, 2015[3]) rules out the possibility that the harmonic pattern observed in laboratory-measured values of G_N is due to some long-range modification of the currently accepted laws of gravitational interaction. This analytical approach may guide future investigations of the systematic uncertainties that plague measurements of G_N.

Based on Symbolic Gauge Theory (SGT), a formalism applied to General Relativity (GR) by R. Mignani, E. Pessa and G. Resconi [4,5] and further developed by I. Licata and G. Resconi, M.E. Rodrigues and E.Koorambas [8,9,10], the E. Koormabas and G. Resconi recently proposed a Non-Conservative Theory of Gravity (NCTG) which can explain the observed variations of G at a 5.9-year scale [9].

The strength of the gravitational force depends on the scale at which the gravitational force is measured by Cavendish-type experiments where two masses (one of which is a test mass) are precisely known, or by (equivalent in principle) gravitational scattering experiments [11]. At laboratory scales, the strength of gravity is characterized by the reduced Planck mass M_{pl} = 2.435 × 10^{18} GeV, which determines Newton's constant $G_N = M_{pl}^{-2}$. Conventionally, the Planck scale M_{Pl} is interpreted as the fundamental scale at which quantum gravitational effects become important in nature. Like all other interactions in nature, nevertheless, the effective strength of gravity is affected by quantum corrections. This effect depends on the characteristic energy of the process probing gravitational interactions (see [12,13] for reviews of an effective theory of gravity). Potential problems of running gravitational couplings by focusing only on physically observable quantities (e.g. amplitudes, cross sections) are discussed in [14,15]. New approaches to the physics of particles with masses greater than 1TeV could offer insights to the problem of the variation of measured G_N values. In such models there is no hierarchy problem [16], whereas quantum gravity can be assessed through experiments at TeV energy levels. That this can be the case in extra-dimensional models is already established [17,18]. Is such modification of gravity also possible in four dimensions [19,39]. Current data from the Large Hadron Collider (LHC) experiments at the European Laboratory for Particle Physics (CERN) do not confirm that gravity becomes stronger around 1 TeV [40-44].

Recently, E.Koorambas suggested that if the observed harmonic pattern of the laboratory-measured values of *G* is due to some environmental or theoretical errors, these errors must also affect the true value of momentum k transferred by the graviton in scattering experiments at the LHC [45]. Furthermore, environmental or theoretical errors could shift the scale of Quantum gravity at 100TeV. Quantum gravity can be investigated by a 100 TeV Proton-Proton Collider as long as environmental or theoretical errors are present. This proposition may explain the current null results for black hole production at the LHC [45].

Although our world appears to consist of 3+1 dimensions (three dimensions of space; and time), it is possible that other dimensions exist, and that these appear at higher energy scales. From the point of view of physics, the concept of extra dimensions received great attention after Kaluza's proposition, in 1921[48], to unify electromagnetism with gravity by identifying the extra components of the metric tensor with the usual gauge fields.

No experimental or observational signs of extra dimensions have been reported. Many theoretical techniques for detecting Kaluza–Klein resonances by means of mass couplings of such resonances with the top quark have been proposed. However, until the LHC reaches full operational power, observation of such resonances is unlikely. An analysis of results from the LHC in December 2010 severely constrains theories with large extra dimensions. [49]

The observation of a Higgs-like boson at the LHC establishes a new empirical test that can be applied to the search for Kaluza–Klein resonances and supersymmetric particles. The loop Feynman diagrams for the Higgs interactions allow any particle with electric charge and mass to run in such a loop. Standard Model particles other than the top quark and W boson do not make large contributions to the cross-section observed in the H → γγ decay. However, if there are new particles beyond the Standard Model, they could potentially change the ratio of the predicted Standard Model H → γγ cross-section to the experimentally observed cross-section. A measurement of any dramatic change to the H → γγ cross-section predicted by the Standard Model is, therefore, crucial in probing the physics beyond it.

Another, more recent, paper, from July 2018 [50], bodes some hope for this theory: in this, the authors dispute that gravity is leaking into higher dimensions as in brane theory. However, the paper does demonstrate that electromagnetism and gravity share the same number of dimensions. This lends support to Kaluza–Klein theory, regardless of whether the number of dimensions is 3+1 or in fact 4+1. The number of dimensions is subject to further debate.

The main aim of the five-dimensional Kaluza–Klein theory is the unification of gravity and electromagnetism. A feature of unification theories was the relation between the electromagnetic coupling e^2 and gravitational coupling G_N, and the radius of the fifth dimension R_c. The radius of the fifth dimension R_c is thus fixed by the elementary electric charge. From the known value of the elementary charge, we find that R_c is of the order of the Planck length. Based on the five-dimensional Kaluza–Klein theory[48,51,52], we show that if the observed harmonic pattern of the laboratory-measured values of G_N is due to some environmental or theoretical errors, these errors must also affect the elementary electric charge e. We then calculate the values of fundamental electric charge e predicted by 3+1 and 4+1 dimensional space-time models. We find that, in the case of 4+1 space-time dimensions, the fundamental electric charge e values oscillate with the 5.9 year LOD oscillation cycle. In In the case of 3+1 space-time dimensions, however, the fundamental electric charge e is constant. Furthermore, we propose that the number of space-time dimensions can be revealed by the 5,9 years repeated Millikan's oil drop experiments.

2. Extra dimensions hypothesis

The initial theory has five-dimensional general coordinate invariance. However, it is assumed that one of the spatial dimensions compactifies, so as to have the geometry of a circle S^1 of very small radius [48,51,52]. Then, there is a residual four-dimensional general coordinate invariance, and, an Abelian gauge invariance associated with transformations of the coordinate of the compact manifold, S^1[48,51,52]. Put another way, the original five-dimensional general coordinate invariance is breaks spontaneously in the ground state. In this way, we arrive at an ordinary theory of gravity in four dimensions and a theory of an Abelian gauge field A_μ. The parameters of the two theories are connected because both theories derive from the same initial five-dimensional Einstein gravity theory [48,51,52].

We adopt the coordinates x^m, m = 1, 2,5 with

$$x^m = (x^\mu, x^5) \quad , \tag{1}$$

where

$$\bar{x}^{\mu} = x^{\mu}, \mu = 0,1,2,3 \tag{2}$$

being coordinates for ordinary four-dimensional space-time, and

$$\bar{x}^{5} = \theta \tag{3}$$

being an angle to parametrize the compact dimension with the geometry of a circle S^1.

The line element is given by

$$d\bar{s}^2 = \bar{g}_{mn} dx^m dx^n \quad , \tag{4}$$

where m,n=1,2,..,5,and $\bar{g}_{mn}$ is the five-dimensional metric.

The five-dimensional Einstein equations yields the following results:

a) For $\bar{g}_{\mu\nu} = g_{\mu\nu}$, the four--dimensional Einstein equations for gravity;
b) For $\bar{g}_{\mu 5} = A_{\mu}$, the Maxwell equations for electromagnetism;
c) For $\bar{g}_{55} = \phi(x)$, the Klein-Gordon equation.

A feature of these theories is the relation between electromagnetic coupling, e^2, gravitational coupling, G_N and p^5, the momentum of the particle in the fifth dimension:

$$e^2 = 16\pi G_N (p^5)^2 . \tag{5}$$

We can use very simple arguments from quantum theory to show that the electric charge is *quantized*, i.e. that *q* is a multiple of some elementary charge *e*. By applying the old Bohr–Sommerfeld quantization rule to the periodic motion, $2\pi r (p_5)_0 = 2\pi n\hbar$, we deduce that, $(p^5)_0 = (n / R_c)\hbar$, which implies that

$$q_n^2 = n^2 e^2 = n^2 \hbar^2 \frac{16\pi G_N}{R_c^2} \quad . \tag{6}$$

The radius of the fifth dimension R_c is thus fixed by the elementary electric charge. From the known value of the elementary charge, we find that R_c is of the order of the Planck length:

$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}, \qquad R_c = \frac{2}{\sqrt{\alpha}} \sqrt{\frac{2\pi G_N \hbar}{8\pi c}} = 3.7 \times 10^{-32} cm \tag{7}$$

If we could calculate the radius R_c from some other considerations, this relation might be used to *calculate* the electric charge. The idea of calculating the elementary electric charge has attracted physicists' attention for a long time, but no satisfactory solution has yet been proposed.

Many years ago [53,54], it was pointed out that the field equations of N = 1 supergravity in d = 11 dimensions admit vacuum solutions corresponding to AdS x S^7, and that, since S^7 admits 8-Killing spinors, and since its isometry group is SO(8), this gives rise (via a Kaluza—Klein mechanism) to an effective d = 4 theory with N = 8 supersymmetry and local SO(8) invariance. There is now a considerable literature on S^7 compactification of d = 11 supergravity [53-59]. An up to date account, paying particular attention to the Brout—

Englert--Higgs—Kibble spontaneous symmetry breaking interpretation of the different S^7 solutions, [57].

In the case of S^7 compactification of d = 11 supergravity, $R_c = m^{-1}$ is just the S^7 radius. However, for more complicated geometries one must be more precise about the meaning of R_c. Weinberg [58] has shown how this is done for an arbitrary geometry with Killing vectors in terms of appropriate root-mean-square circumferences. The precise constants of proportionality in (5) depend crucially on the field content of the higher-dimensional theory. Although at the classical level the "size" of S^7 is undetermined, Candelas and Weinberg [59] have pointed out that in a certain class of theories admitting a compactification due to one-loop radiative corrections one may calculate R, and hence, in a realistic theory, the fine structure constant $\alpha = e^2 / 4\pi$.

More recently, it has been realized that the hierarchy problem could be addressed, and possibly solved, by utilising the geometry of space-time. In many extra-dimensional models, the (3 + 1)-dimensional space time we experience is a structure called a brane, which is embedded in a (3 + k + 1) space time called the bulk. The hierarchy problem can then be addressed by postulating that all extra dimensions are compactified on circles (or other topology) of some size, R (as has been done in the Arkani Hamed, Dimopoulos and Dvali (ADD) scenario [60]), thus lowering the fundamental Planck scale to an energy near the electroweak scale. Alternatively, this could be accomplished by introducing extra dimensions with large curvature (warped extra dimensions), as has been suggested by Randall and Sundrum [61]. The extra dimensional scenario which we will focus on in the remainder of this review (universal extra dimensions) does not share the features of the ADD or RS scenarios. Instead, it introduces flat extra dimensions which are much smaller than those in the ADD framework.

In addition to the hierarchy problem, motivation for the study of theories with extra dimensions comes from string theory and M-theory, which today appear to be the best candidates for a consistent theory of quantum gravity and a unified description of all interactions. It appears that such theories may require the presence of six or seven extra-dimensions. A general feature of extra-dimensional theories is that upon compactification of the extra dimensions, all of the fields propagating in the bulk have their momentum quantized in units of $p^2 \sim 1/R^2$. The result is that for each bulk field, a set of Fourier expanded modes, called Kaluza–Klein (KK) states, appears. From our point of view in the four-dimensional world, these KK states appear as a series (called a tower) of states with masses $m_n = n/R$, where n labels the mode number. Each of these new states contains the same quantum numbers, such as charge, color, etc. In many scenarios, the Standard Model fields are assumed to be confined on the brane, with only gravity allowed to propagate in the bulk. Nevertheless, if the extra-dimensions are small, it would be possible for all fields to freely propagate in the extra dimensions. Such is the case in models with universal extra dimensions (UED). Scenarios in which all fields are allowed to propagate in the bulk are called universal extra dimensions UED [62]. Following Ref. [63]. In the case of one extra dimension, the constraint on the compactification scale in UED models from precision electroweak measurements is as low as R^{-1}=300 GeV [62]. Recently, it was shown that this bound can be weakened to R^{-1}=280 GeV if one allows a Higgs mass as heavy as m_H=800 GeV [64]. This is to be contrasted with another class of models where Standard Model bosons propagate in extra dimensions while fermions are localized in 4 dimensions. In such cases, the constraint on the compactification scale is much stronger, requiring R^{-1}= 1 TeV [65].

The prospect of UED models providing a viable dark matter candidate is indeed what motivates us in our discussion here. The existence of a viable dark matter candidate can be seen as a consequence of the conservation of momentum in higher dimensional space. Momentum conservation in the compactified dimensions leads to the conservation of KK number. However, this does not stabilise the lightest KK state. To generate chiral fermions at the zero mode, the extra dimensions must be modeled out by an orbifold, such as S/Z_2 for one extra dimension or T_2/Z_2 for two. This orbifolding results in the violating of KK number, but can leave a remnant of this symmetry called KK-parity (assuming that the boundary terms match). All odd-level KK particles are charged under this symmetry, thus ensuring that the lightest (first level) KK state is stable. In this way, the lightest Kaluza–Klein particle (LKP) is stabilized in a way quite analogous to the LSP in R-parity conserving supersymmetry. In the next section, we will discuss some of the characteristics of the LKP in models of UED.

3. The sinusoidal variations of Newton's coupling constant

Measurements of the gravitational constant (G) are notoriously difficult due to the gravitational force being by far the weakest of the four known forces. Recent advances, making use of electronically controlled torsion strip balances at the *Bureau International des Poids et Mesures* (BIPM) in the last 15 years, have improved the accuracy of G measurements (see [20] for details on experimental methods). These recent measurements have also revealed a peculiar type of oscillatory variation, seemingly following a 5.9 years cycle akin to the so called Length-of-Day (LOD) [1].

Although we recognize that the correlation between G measurements and the 5.9 year LOD cycle could be fortuitous, we think that this is unlikely, given the striking match between these two (Fig. 1).

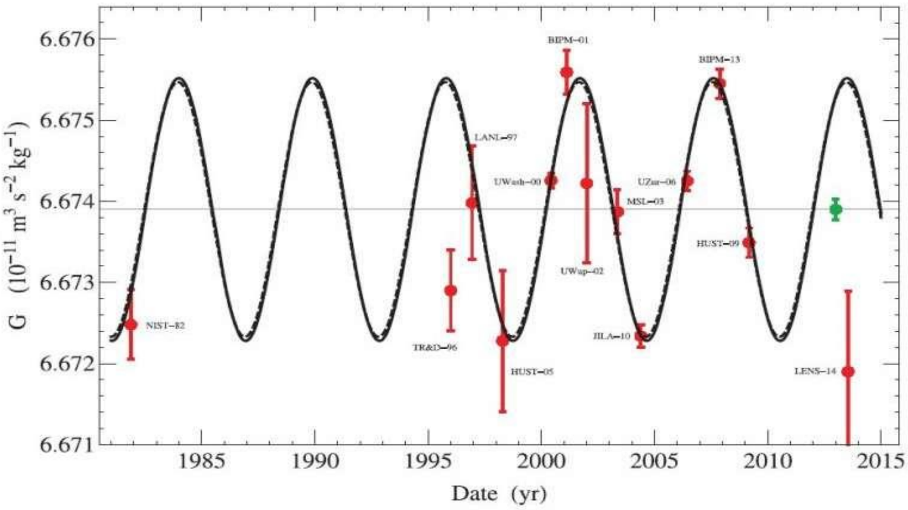

Fig. 1: Comparison of the CODATA set of G measurements) with a fitted sine wave (solid curve) and the 5.9 year oscillation in LOD daily measurements (dashed curve), scaled in amplitude to match the fitted G sine wave. Acronyms for the measurements follow the CODATA convention. Also included are a relatively recent BIPM result from Quinn *et al.* [21] and measurement LENS-14 from the MAGIA collaboration [22] that uses a new technique of laser-cooled atoms and quantum interferometry, rather than the macroscopic masses of all the other experiments. The green filled circle represents the weighted mean of the included measurements, along with its one-sigma error bar, determined by minimizing the L1 norm for all 13 points and taking into account the periodic variation.

The observed correlation cannot be due to centrifugal force acting on the experimental apparatus, since changes in LOD are too small by a factor of about 10^5 to explain changes in G. This is because the Earth's angular velocity ω_E is by definition

$$\omega_E = \omega_0 \left(1 - LOD\right), \tag{8}$$

where ω_0 is an adopted sidereal frequency equal to 72921151.467064 prad s^{-1} and the LOD is in ms d^{-1} (www.iers.org). The total centrifugal acceleration is given by:

$$a_c = r_s \omega_0^2 \left[1 - 2A\sin\left(\frac{2\pi}{P}(t - t_0)\right)\right], \tag{9}$$

where A is the amplitude of the 5.9 year sinusoidal LOD variation (= 0.000150/86400), and r_s is the distance of the apparatus from the Earth's spin axis. The maximum percentage variation of the LOD term is 3.47×10^{-9} of the steady-state acceleration, while dG/G is $2.4{\times}10^{-4}$. Even the full effect of the acceleration with no experimental compensation changes G by only 10^{-5} of the amplitude shown in Fig. 1.

Following Anderson et al. 2015a [1], the shift from the true value of renormalized gravitational constant is given by:

$$\begin{aligned} G(t)_{ren}^{shift} &= G_{ren} + \delta G(t)_{ren}^{Error} = G_{ren} + B_G \sin(a_G t + \varphi) \\ &= G_{ren} + 2G_{ren} A_G \sin(a_G t + \varphi), \end{aligned} \tag{10}$$

where

$$B_G = 2G_{ren} A_G \,, \tag{11}$$

and

$$A_G = 10^{-4}, \varphi = 80.9\ deg, a_G = 2\pi / P_G, P_G = 5.899\ yr. \text{ (Anderson et al. 2015a) [1].} \tag{12}$$

Here, the variation term due to environmental or theoretical errors $\delta G(t)_{ren}^{Error}$ in equation (10) is given by

$$\begin{aligned} &\delta G(0,t)_{ren}^{error} = G_{ren} f(t)^{error}, \\ &f(t)^{error} = 2A_G \sin(a_G t + \varphi) \end{aligned} \tag{13}$$

where G_{ren} is the renormalized gravitational constant and $f(t)_{error}$ is the environmental or theoretical error.

4. Gravitational action in the presence of environmental or theoretical errors

A scalar-tensor theory of gravity (STG), first proposed by Brans and Dicke [66], was inspired by a suggestion of Dirac's that the gravitational constant G_N varies with time [67]. In the Scalar-tensor theories of gravity, the gravitational action can be written:

$$S = \int d^n x \sqrt{-g} \left(-\frac{R}{16\pi G_N} f(\phi) + \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) + F[\phi, g_{\mu\nu}] \right) \text{ [72]} \tag{14}$$

What characterizes different STG models is the specific choice of f(φ), V (φ) and F [φ, $g_{\mu\nu}$], a local scalar function of φ, $g_{\mu\nu}$ and their derivatives. The coefficient of the Ricci scalar R in conventional General Relativity (GR) is proportional to the inverse of Newton's constant G_N [66-72]. In scalar-tensor theories, then, where this coefficient is replaced by some function of a field which can vary throughout space-time, the "strength" of gravity (as measured by the local value of Newton's constant G_N) will be different from place to place and time to time [66-72].

Following my previous paper [45], we propose a scalar-tensor gravity where the scalar field is the environmental or theoretical error $f(t)_{error}$ given by equation. (13). In this proposition, the variation of gravity due to environmental or theoretical errors is given by:

$$\delta_G S=\int d^n x\sqrt{-g}\left(-\frac{R}{16\pi G_N}F(t)_{error}+\frac{1}{2}g^{00}(\partial_0 f(t)_{error})(\partial_0 f(t)_{error})\right) \\ =\int d^n x\sqrt{-g}\left(-\frac{(-1)^N R}{16\pi\delta G_N(t)^+ + N}\right), \qquad (15)$$

where

$$(\partial_0 f(t)_{error})(\partial_0 f(t)_{error})=4a_G^2 A_G^2\cos^2(a_G t+\varphi), \quad \text{(vanish by equation.12)}, \qquad (16)$$

and

$$F(t)_{error}=\frac{(-1)^N}{|f(t)_{error}|+\left(L_{pl}^2/1cm^2\right)N}, \qquad N=0,1,2,3,...... \qquad (17)$$

In equation.17, L_{pl} is the Planck length and *N* are the integers.

Without any loss of generality, we assume that the variation of the gravitational constant $\delta G_N(t)^{+(error)}$ in the action (15), defined by the absolute value of the function $|f(t)_{error}|$ [45] is:

$$\delta G_N(t)^{+(error)}G_N\,|f(t)_{error}|=2A_G G_N\,|\sin(a_G t+\varphi)|=\frac{\delta g(t)^2}{M_{Pl}^2}, \qquad (18)$$

where

$$\delta g(t)^2=2g^2 A_G\,|\sin(a_G t+\varphi)|, \qquad (19)$$

$$G_N=\frac{g^2}{M_{Pl}^2}. \qquad (20)$$

In equation18, the variation of gravitational constant δG_N is absorbed by the dimensionless gravitational coupling δg given by equation.19. This differs from is my previous paper [45] where g was considered as a constant and the variation of the gravitational constant δG_N was inversely proportional to the variation of the square Plank mass δM_{Pl}^2.

From action (15), we obtain the gravitational action in the presence of environmental or theoretical errors:

$$S' = S_{EH} \mp \delta_G S\,, \tag{21}$$

where

$$S_{EH} = \int d^n x\sqrt{-g}\left(-\frac{R}{16\pi G_N}\right), \tag{22}$$

S_{HE} is the Einstein-Hilbert action; $\delta_G S$ and $f(t)_{error}$ are as in equation 13.

The zeros of the error function $f(\theta(t))_{error} : \theta(t) = a_G t + \varphi$ (equation13) are calculated as follows:

If $f(\theta(t))_{error} = 2A_G \sin\theta(t)_{error} = A_G \dfrac{e^{i\theta(t)_{error}} - e^{-i\theta(t)_{error}}}{i} = 0$, (23)

then

$$e^{i\theta(t)_{error}} = e^{-i\theta(t)_{error}} \text{ or } e^{2i\theta(t)_{error}} = 1 = e^{2k\pi i}\,,\ k = 0, \pm 1, \pm 2, \ldots \tag{24}$$

Hence $2i\theta(t)_{error} = 2k\pi i$ and $\theta_{error}^{zeros}(t) = k\pi = 0, \pm\pi, \pm 2\pi, \ldots$, i.e. the latter are all zeros and real.

Now, we calculate the action.15 at zeros $\theta(t) = \theta_{error}^{zeros}(t)$ for time scale *t<<P*$_G$*=5,8yr*, when $N \to \infty$, as follows:

$$\begin{aligned} \lim_{\substack{\theta\to\theta^{(zero)}\\ N\to\infty}} \delta_G S &= \lim_{\substack{\theta\to\theta^{(zero)}\\ N\to\infty}} \left[\int d^n x\sqrt{-g}\left(-\frac{(-1)^N R}{16\pi\delta G_N(t)^+ + N}\right)\right] \\ &= -\lim_{\substack{\theta\to\theta^{(zero)}\\ N\to\infty}} \left(\frac{(-1)^N}{16\pi\delta G_N(t)^+ + N}\right)\int d^n x\sqrt{-g}R = -\lim_{N\to\infty}\left(\frac{(-1)^N}{N}\right)\int d^n x\sqrt{-g}R = 0 \end{aligned}, \tag{25}$$

where

$$\delta G_N(t)^+ \to 0\ ,\ \theta_{error}(t) \to \theta_{error}^{(zeros)}(t) \text{ (by equations.13, 23)}, \tag{26}$$

and

$$\lim_{N\to\infty}\left(-\frac{(-1)^N}{N}\right) = 0\,. \tag{27}$$

Using equations.13, 17-19, and the interactions of graviton with matter in the presence of environmental or theoretical errors, it can be written a:

$$\delta_G \Im_h^{\text{int}} = -\left(\frac{(-1)^N}{16\pi\delta G_N(t)^+ + N}\right) h_{\mu\nu} T^{\mu\nu}, \tag{28}$$

where $h_{\mu\nu}$ is the graviton and $T^{\mu\nu}$ is the energy-momentum tensor. With the requirement that non-renoremalizabele terms be suppressed by inverse powers of Planck mass (equation.20), the propose theory will become renormalize in the limits.26, 27: $\delta G_N(t)^+ \to 0$, $M_{Pl}^2 \to \infty$ when $N \to \infty$. (for details see [45]) Indeed, in the presence of environmental or theoretical errors, we treat quantum gravity as an effective theory by taking the limit $M_{Pl}^2 \to \infty$ (as discussed in [45]).

5. Kaluza-Klein gravity in the presence of Newton's constant variation δG$_N$ in (4+1) space-time dimensions

Following Ref. [52]. We adopt coordinates x^A, A = 1, 2, 5, with respects to equations.1-3. After compactification, the ground-state metric is:

$$\tilde{g}_{AB}^{(0)} = diag\left\{\eta_{\mu\nu} - \tilde{g}_{55}\right\}, \tag{29}$$

Where

$$\eta_{\mu\nu} = (1, -1. -1, -1), \tag{30}$$

is the metric of Minkowski space, M_4, and

$$\tilde{g}_{55} = R_c^2 \tag{31}$$

is the metric of the compact manifold S^1, where R_c is the radius of the circle. The identification of the Abelian gauge field $B_\mu(x,\theta)$ arises from an expansion of the metric about the ground state. We parametrize the metric in the form:

$$\tilde{g}_{AB}(x,\theta) = \begin{pmatrix} g_{\mu\nu}(x,\theta) - B_\mu(x,\theta)B_\nu(x,\theta)\Phi(x,\theta) & B_\mu(x,\theta)\Phi(x,\theta) \\ B_\nu(x,\theta)\Phi(x,\theta) & -\Phi(x,\theta) \end{pmatrix} \tag{32}$$

To extract the graviton and the Abelian gauge field, it is sufficient to replace $\Phi(x,\theta)$ with its ground-state value $\tilde{g}_{55}$ and to use the ansatz without θ dependence:

$$\tilde{g}_{AB}(x) = \begin{pmatrix} g_{\mu\nu}(x,\theta) - B_\mu(x)B_\nu(x)\tilde{g}_{55}(x) & B_\mu(x)\tilde{g}_{55}(x) \\ B_\nu(x)\tilde{g}_{55}(x) & -\tilde{g}_{55}(x) \end{pmatrix}. \tag{33}$$

We write

$$B_\mu(x) = \xi A_\mu(x), \tag{34}$$

where ξ is a scale factor, to be chosen so that $A_\mu(x)$ is a conventionally normalized gauge field.

Coordinate transformations associated with the coordinate θ of the compact manifold may be interpreted as gauge transformations, as we proceed to show. Consider the transformation:

$$\theta \to \theta' = \theta + \xi\varepsilon(x) \tag{35}$$

For a general coordinate transformation

$$\tilde{g}_{AB} = \tilde{g}'_{A'B'} \frac{\partial \tilde{x}'^{A'}}{\partial \tilde{x}^{A'}} \frac{\partial \tilde{x}'^{B'}}{\partial \tilde{x}^{B}} \quad . \tag{36}$$

For the particular transformation (35), the off-diagonal elements of the metric give

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \varepsilon \quad . \tag{37}$$

Transformation (35) of the coordinates of the compact manifold, therefore, induces an Abelian gauge transformation on $A_\mu(x)$**.** This means that the compact manifold provides the internal symmetry space for the Abelian U (1) gauge group. Internal symmetry has now to be interpreted as just space-time symmetry, but associated with the extra spatial dimension.

An effective action for the four-dimensional theory may be derived from the action for five-dimensional Einstein gravity in the presence of environmental or theoretical errors. Following equations.15-20, we have the five-dimensional action:

$$\delta_G S_g = \int d^5x \,|\det \tilde{g}\,|^{1/2} \left(-\frac{\tilde{R}}{16\pi\tilde{G}_N} F(t)_{error} \right) = \int d^5x \,|\det \tilde{g}\,|^{1/2} \left(-\frac{(-1)^N \tilde{R}}{16\pi\delta\tilde{G}_N(t)^+ + N} \right) \tag{38}$$

$\tilde{R}$; is the five-dimensional curvature scalar, $F(t)_{error}$ is given by equation.17 and $\tilde{G}_N$, $\delta\tilde{G}_N(t)^+$ are the gravitational constant for five dimensions and its variation, respectively.

Substituting ansatz (33) for $\tilde{g}_{AB}$**,** and integrating over the extra spatial coordinate θ**,** gives an effective four-dimensional action in the presence of environmental or theoretical errors for time scales *t<<PG=5,8yr*:

$$\delta_G S = -\frac{2\pi(-1)^N R_c}{16\pi\delta\tilde{G}_N(t)^+ + N} \int d^5x \,|\det g\,|^{1/2}\, R - \frac{\delta\xi^2 \tilde{g}_{55}}{4} \left(\frac{2\pi(-1)^N R_c}{16\pi\delta\tilde{G}_N(t)^+ + N} \right) \times \int d^5x \,|\det g\,|^{1/2}\, F_{\mu\nu}F^{\mu\nu}, \tag{39}$$

where R_c is the radius of the compact manifold as in equation.31, R is the four-dimensional curvature scalar, and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \tag{40}$$

Now, the variation of the four-dimensional gravitational constant δG_N becomes

$$\delta G_N(t)^+ = \frac{\delta\tilde{G}_N(t)^+}{2\pi R_c} \quad . \tag{41}$$

To obtain standard normalization for the gauge field, we must then choose

$$\delta\xi^2(t) = \frac{16\pi\delta G_N(t)^+}{\tilde{g}_{55}} = \frac{\delta\kappa^2(t)}{R_c^2} \quad , \tag{42}$$

where

$$\delta\kappa^2(t) = 16\pi\delta G_N(t)^+ . \tag{43}$$

Then, the effective four-dimensional action in the presence of environmental for time scales *t<<P_G=5,8yr* is given by

$$\delta_G S = -\frac{(-1)^N}{16\pi\,|\,\delta G_N(t)\,|+N}\int d^5x\,|\det g\,|^{1/2}\,R - \frac{1}{4}\int d^5x\,|\det g\,|^{1/2}\,F_{\mu\nu}F^{\mu\nu} \tag{44}$$

5.*1. The electron's unit charge from the fifth dimension*

The natural scale of mass for these theories is the Planck mass M_{pl}. Massive fields in five dimensions will naturally result in to particles with masses on the Planck scale in four dimensions. Suppose, instead, that we start with a massless field in five dimensions. For a five-dimensional scalar field Φ(x,θ), we may make the Fourier expansion on the compact manifold:

$$\phi(x,\theta) = \sum_{n=-\infty}^{\infty}\phi^n(x)e^{in\theta} \quad . \tag{45}$$

The Klein-Gordon (KG) equation,

$$\left(\Box_x - \frac{1}{R_c^2}\frac{\partial^2}{\partial\theta^2}\right)\phi(x,\theta) = 0, \quad , \qquad \Box_x = g^{\mu\nu}\partial_\mu\partial_\nu = \partial^\mu\partial_\nu \tag{46}$$

then gives the equations for the Fourier components:

$$\left(\Box_x + M_n^2\right)\phi^n(x) = 0 \quad , \tag{47}$$

where

$$M_n^2 = \frac{n^2}{R_c^2}, \qquad n = 0, \pm1, \pm2, \pm3, ... \tag{48}$$

The fields Φ^n(x) are thus the mass eigenstates in four dimensions, and the field Φ(x) is the only massless one or perhaps light, after allowing for radiative corrections[52]. The other fields Φ^n(x) have masses of the order R_c^{-1}, which we would expect to be comparable to the Planck mass M_{pl}. We see that all scalar particles have ***n*** ≠ 0. But, from (48), this means that they all have masses at the Planck scale M_{pl}, whereas the familiar particles have very small masses at that scale. In this way Klein explained (for the first time) the quantization of electric charge [51]. (Note also that charge conjugation is just parity transformation y → −y in the fifth dimension). Of course, if we identify the fundamental unit of charge ($e = \sqrt{216\pi G_N M_c}$ as in equation.6) with the charge of the electron, then we are forced to take M_c to be very large: the Planck mass 10^{19} GeV, well beyond the range of any current or foreseeable accelerator. This answers the second question left unanswered by Kaluza: with M_c being very large, the radius of the circle must be very small, at the order of the Planck size (10^{-35} meters), which accords satisfactorily with our everyday experience of living in four space-time dimensions [73].

5.2. On the variations of electron's unit charge in the presence of environmental or theoretical errors in five-dimensional space-time

Following Ref. [52]. If we apply the coordinate transformation.35 in the presence of environmental or theoretical errors,

$$\theta \to \theta' = \theta + \delta\xi(t)\varepsilon(x) \tag{49}$$

to the $\phi(x,\theta)$ as in equation. (58), we have:

$$\phi(x) \to \exp\left(i\delta\xi(t)\varepsilon(x)\right)\phi(x) \tag{50}$$

Here, $\delta\xi(t)$, given by equation.42, is treated as a constant for time scales $t<<P_G=5,8yr$.

The Abelian gauge field transforms.37 in the manner,

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \varepsilon \quad . \tag{51}$$

This means that $\phi(x)$ has the variation of the unit electric charge $\delta\boldsymbol{e}$ due to the presence of environmental or theoretical errors:

$$\delta e(t) = -\delta\xi(t) = \frac{\delta\kappa(t)}{R_c} \quad , \tag{52}$$

where we have used the normalization condition.42. Thus, the variation of electric charge $\delta e^2(t)$ is quantized in units of $\delta\kappa(t)/R_c$**.** Using equation.43, the variation of the unit electric charge δe^2 can be written as;

$$\delta e^2(t) = \frac{16\pi\delta G_N(t)^+}{R_c^2} \quad . \tag{53}$$

Inserting equations.13 to 53, and using equation.6, the variation of elementary electric charge can be expressed as follows:

$$\delta e^2(t) = 2A_G e^2 \,|\sin(a_G t + \varphi)|\,. \tag{54}$$

By using equation. (54), we obtain the time depended unit of charge of the electron in the presence of environmental or theoretical errors,

$$e(t) = e + e\sqrt{2A_G\,|\sin(a_G t + \varphi)|}\,, \tag{55}$$

where $A_G = 10^{-4}, \varphi = 80.9\ deg, a_G = 2\pi / P_G, P_G = 5.899\ yr.$ (Anderson et al. 2015a) [1], and e is the electron unit charge.

6. Testing the number of space-time dimensions by the proposed P_G=5,9yr repeated Millikan's oil drop experiments

The oil drop experiment was performed originally by American physicist Robert A. Millikan in 1909[74] to measure the charge of a single electron. The experiment apparatus (Figure.2) consists of an atomizer which sprays tiny oil droplets and of a short focal distance telescope, by means of which the droplets can be viewed. There are two plates, one of positive and one of negative charge, above and below the bottom chamber. A dc supply is attached to the plates. Some of the oil drops fall through the hole in the upper plate. The bottom chamber is illuminated with X-rays that cause the air to ionize. As the droplets traverse through the air, electrons accumulate over the droplets and negative charge is acquired. With the help of the dc supply a voltage is applied. The speed of droplet motion can be controlled by altering the voltage applied on the plates [74-78].

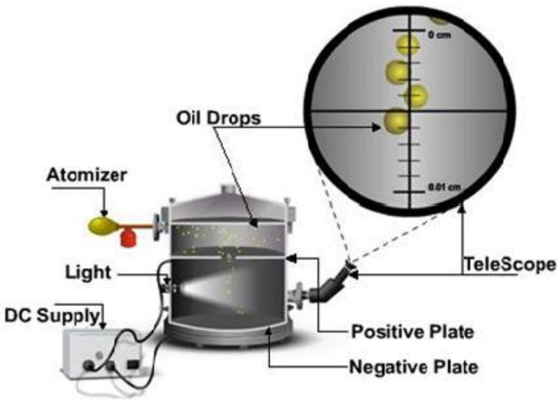

Fig. 2. Design of the Millikan oil-drop experiment for determining the electric charge of the electron.

By adjusting the applied voltage, a droplet can be suspended in the air. Millikan observed one drop after another, varying the voltage and noting the effect. After many repetitions, he concluded that charge could assume only certain fixed values. He repeated the experiment for many droplets and confirmed that the charges were all multiples of some fundamental value and calculated it to be 1.5924(17) $\times 10^{-19}$ C, within one percent of the currently accepted value of 1.602176487(40) $\times 10^{-19}$ C. He proposed that this was the charge of a single electron [74-78]. Millikan's paper [74], presented a complete summary of data on 58 drops studied over 60 consecutive days. Mathematically, Millikan started with the following equation:

$$v_1 / v_2 = mg / Fe - mg \quad . \tag{56}$$

With appropriate substitutions, the equation takes the following form:

$$e_n = (4/3)\pi(9\mu/2)^{3/2}\{1/g(\sigma-\delta)\}^{1/2}(v_1+v_2)v_1^{1/2}/F... \tag{57}$$

Including the correction from Stokes' law gives the equation:

$$v_1 = 2/9ga^2(\sigma-\delta)\mu\{1+A/\alpha\} \tag{58}$$

Combining equations (57) and (58) gives the value of *e*:

$$e(1+A/\alpha)^{3/2}=e_n \qquad (59)$$

where v_1: speed of descent of the drop under gravity; v_2 : speed of ascent of the drop in the electric field; mg : force of gravity; F : electric field; e_n : frictional charge on the drop; μ ;coefficient of viscosity of air; s density of the oil; d = density of air; a = radius of the drop; l : mean free path of a gas molecule; and A : correction term constant. The mean value obtained with this method was reported to be: e =4.774 ±0.009×10^{-10} esu. At this stage, it is important to note that Millikan, based on his guiding assumptions, expected the value of e_n to be an integral multiple of e, where n = 1, 2, 3, . . . Apparently Millikan discarded values that did not turn out to be integral multiples [78]. Note that there is a larger gap between the values 2.2x10^{-19} C and 2.9x10^{-19} C than between the other points that define the first five gaps (increments). We cluster the first six values (e_1 to e_6) together by averaging that group, and we assign that group to integer 1 (1 unit of charge as in equation. 55). The next significant gap occurs between 3.7x10^{-19} C and 4.5x10^{-19} C, so we average the values between 2.9x10^{-19} C and 3.7 x 10^{-19} C into the second cluster and assign them to integer 2 (2 units of charge) [78].

Since the electron unit charge is given by equation.75, we propose repeated Millikan's oil drop experiments with the time scale of 5.9 year, equally to the LOD oscillation P_G=5,899yr, Newtonian constant of gravitation G_N. Substituting equation.55 to 59, we find the variation of electron fundamental charge measured by repeated Millikan's oil drop experiments with time scale equally to P_G=5,899yr:

$$\begin{aligned} e(t)(1+A/\alpha)^{3/2} &= (e+e\sqrt{A_G\,|\sin(a_G t+\varphi)|})(1+A/\alpha)^{3/2} \\ &= e_n\left(1+\sqrt{2A_G\,|\sin(a_G t+\varphi)|}\right)=e_n(t) \end{aligned} \qquad (60)$$

for five dimensional space-time;

$$e(1+A/\alpha)^{3/2}=e_n \qquad (61)$$

for four dimensional space-time,

with $A_G=10^{-4}, \varphi=$ 80.9 $deg, a_G=2\pi/P_G, P_G=$ 5.899 $yr.$ (Anderson et al. 2015a) [1], and e being of the currently accepted value of 1.602176487(40) ×10^{-19} C of electron charge [78].

The comparison between the values of fundamental electric charge e predicted by four dimensional space-time equation. (61), and the set of e(t) values predicted by the five dimensional space-time model (60) with the 5.9 year LOD oscillation cycle is calculated in Table.1 and shown in Fig.3.

t (years)	e(Coulomb) predicted by 4+1 dimensions Eq.(60)	e(Coulomb) predicted by 3+1 dimensions Eq.(61)
0	1.62469×10^{-19}	1.602176487(40)×10^{-19}
1	1.61997×10^{-19}	1.602176487(40)×10^{-19}
2	1.61633×10^{-19}	1.602176487(40)×10^{-19}
3	1.62477×10^{-19}	1.602176487(40)×10^{-19}

4	1.61933×10^{-19}	$1.602176487(40)\times10^{-19}$
5	1.61719×10^{-19}	$1.602176487(40)\times10^{-19}$
6	1.62482×10^{-19}	$1.602176487(40)\times10^{-19}$

Table.1. The first column is the time decimal in years. The second column is the fundamental electric charge e(t) predicted by the five dimensional space-time model (60). The third column is the electric charge e predicted by the four dimensional space-time equation. (61).

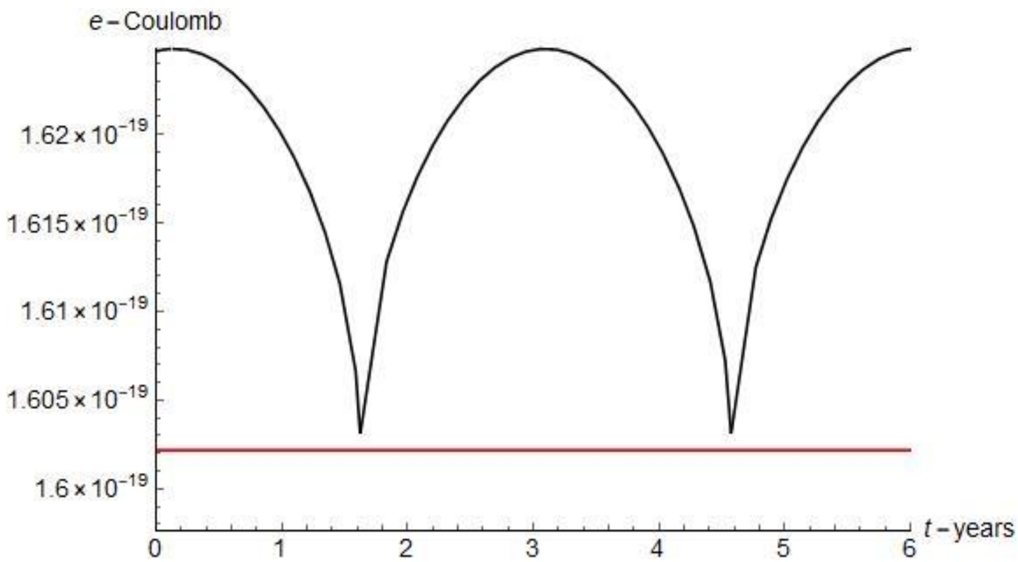

Fig. 3 Comparison between the values of fundamental electric charge e predicted by four dimensional space-time, and the set of e(t) values predicted by the five dimensional space-time model with the 5.9 year LOD oscillation cycle. The red curve shows the values of e predicted by the four dimensional space-time equation. (61); the black curve is the theoretical variation of e(t) predicted by five-dimensional model (60).

7. Discussion

In 2014, the UK's Royal society hosted a conference titled "*The Newtonian Constant of Gravitation, a constant too difficult to measure?*" [79], which was intended to resolve the problem of large discrepancy between recent G_N values. [80]

Another reasonable explanation for the discrepancy of G_N measurements is that there is still some unknown physical cause. [80,45,46] In 2015, Anderson et al. claimed that the recent values of G_N varied sinusoidally with a period of about 5.9 years by analyzing the measurement results [1], and they proposed that one possible reason for this variation was the activity of the Earth's core. Then Schlamminger et al. corrected the acquisition time of these measurement results but did not find any remarkable correlation [46]. In 2017, Parra proposed that the temporal variation of G_N was potentially caused by the sun's dragging effect. [81] These hypotheses can be neither confirmed nor refuted at present, since the precision of G_N

measurement is low. G_N measurements of higher precision, obtained by more methods, are, therefore, required.

Using the normalization condition.42, we find that the variation of electric charge $\delta e^2(t)$ is quantized in units of $16\pi\delta G_N(t)/R_c$ and is proportional- to the sinusoidally variation of dG_N with a period of about 5.9 years (Table.1; Figure. 3). We also observe that the changes of electron charge due to the presence of a fifth space dimension should be about 10^{-2} with a period of about 5.9 years (as shown by the red curve in Figure 3). In the case of four space-time dimensions, the electron charge is constant and perfectly fits a straight line (black curve in Fgiure 3). We propose that the number of space-time dimensions can be revealed by the 5,9 years repeated Millikan's oil drop experiments.

8. Conclusion

The core aim of the five-dimensional Kaluza–Klein theory is the unification of gravity and electromagnetism. A feature of unification theories is the relation between the electromagnetic coupling e^2, gravitational coupling G_N, and the radius of the fifth dimension R_c. The radius of the fifth dimension R_c is thus fixed by the elementary electric charge. From the known value of the elementary charge, we find that R_c is of the order of the Planck length. Based on the five-dimensional Kaluza–Klein theory, we show that, if the observed harmonic pattern of the laboratory-measured values of G_N is due to some environmental or theoretical errors, these errors must also affect the elementary electric charge e. We calculate the values of fundamental electric charge e predicted by 3+1 and 4+1 dimensional space-time models. We find that, in the case of 4+1 the fundamental electric charge, e values oscillate with the 5.9 year LOD oscillation cycle, while in the case of 3+1 space-time dimensions the fundamental electric charge e is constant. We also propose that the number of space-time dimensions can be revealed by the 5,9years repeated Millikan's oil drop experiments.

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