Proof of Goldbach's strong conjecture

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Abstract: Proof of the conjecture by unique path method

Please note: Every symbol 'p' with or without any suffix or (*) denotes some prime number. a|b means a divides b and a®b means a doesn't divide b. n is a natural number. The word 'prime' would hereafter mean prime number. The sign '∃' means 'there exist'.

First stage: There is at least one prime p (3≤p<n) for every 2n>6 such that p®2n.
Proof: Consider the evens 2n-2 or 2n+2 such that they are not integral powers of 2. Now n-1 or n+1 is divisible by at least one prime p* (3≤p*<n).
So p*2(n+1)⇒p*®2(n+1)-2⇒p*®2n
Second stage: Consider a prime p₁ (3≤p₁<n) such that p₁®2n. Now let 2n-p₁ is divisible by a prime p₂, where p₂<n.
So there can be a series
∃p₂, such that p₂|2n-p₁, where p₂<n
∃p₃, such that p₃|2n-p₂, where p₃<n
...
... ...
...
... ...
∃p_k, such that p_k|2n-p_k-1, where p_k<n.
The primes p₂, p₃,... are taken in such a manner as far as possible that each one is different from all the previous ones in the list of primes in the L.H.S of 'such that'.
It can easily be proved that no such prime divides 2n (since any p_k≠p_k-1).
The operation of getting p₂,p₃,... must end at some p_k, otherwise there will be infinite number of primes<n. We henceforth shall call p₁ or any prime (which is <n and doesn't divide 2n) functioning like it at the very start of the R.H.S as in the above series, as 'starting prime' and association of the steps starting with it and ended with the last available different prime in the list of primes in the L.H.S of 'such that' as above (like p_k, as termed in the above case), as a system.

We further call p₂,p₃,... as outputs.

Observation (1): For a particular p₁ we can choose arbitrarily particular p₂, p₃,..., p_k (p_k being the last available different output for the system where p₁ is the starting prime) and in this way they constitute an Unique Path of successive particular selections from the prime factors of various 2n-p₁'s, p₁'s starting from p₁, so that the system becomes uniquely sized consisting of unique k-1 steps.
Therefore none of p₂, p₃,..., p_k equals to p₁. Otherwise, the system size would reduce, which contradicts the hypothetically unique path above or there can be p₁|2n (as in the case of k=2, p₂=p₁).

Observation (2): p_k being the last different output (<n) from 2n-p_k-1 in the system, proceeding similarly beyond it we get p_k+1 from 2n-p_k where, p_k+1|2n-p_k. Though in this case p_k+1 may not be necessarily <n.
Now $p_{k+1}<n$ implies $p_{k+1}$ is a recycled prime i.e, $p_{k+1}$ is any of $p_1, p_2, p_3, \ldots, p_k$ (since $p_k$ is the last available different output for the system of $k-1$ unique steps, $p_1$ being the starting prime).

If $p_{k+1}=p_1$. Since $p_1|2n-p_k$, $p_1<n$ and $p_1$ is different from $p_2, p_3, \ldots, p_k$, the system starting with $p_1$ has at least $k$ unique steps instead of only unique $k-1$ steps, increasing its size, which means extension of the unique path and thus gives a contradiction to the same.

So from the above, we can conclude that no starting prime can be recycled as an output at the next step after the last different output obtained in any such system of any size.

Observation (3) : If some $p_r$ of $p_2, p_3, \ldots, p_k$ is recycled, i.e, equals to $p_{k+1}$, then taking $p_r$ as a starting prime and replicating the part of unique path till the predefined end as described in Observation (1) we can get a system of reduced size the next step to the last of which recycles the starting prime $p_r$ as an output, which is impossible according to the last conclusion from Observation (2). [In the case of $k=2$, $p_2$ can never be recycled because $p_{k+1} \neq p_2$]

Summing up the above discussions we are bound to accept the conclusion that $p_{k+1}>n \Rightarrow p_{k+1}=2n-p_k$.

I.e, $2n=p_k+p_{k+1}$

If $p_2>n$, then $2n=p_1+p_2$

Finally $6=3+3$ and $4=2+2$

Therefore Goldbach’s strong conjecture holds for every $2n \geq 4$