Refutation of translation of implicit logic (IL) to explicit logic (EL)

Abstract: For translation of implicit logic to explicit logic, using intuitionistic and epistemic logic, seven equations are evaluated, with none tautologous. Two refute the recursive translation of "the intuitionistic truth definition into a syntactic recipe"; three refute "key features of intuitionistic logic in modal terms"; and one refutes "the recursion law after knowledge update [as] the basic dynamic equation of hard information" for public announcement logic (PAL). These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: van Benthem, J. (2019). Implicit and explicit stances in logic.
Journal of Philosophical Logic. 48:571–601
link.springer.com/content/pdf/10.1007%2Fs10992-018-9485-y.pdf johan@stanford.edu

N.B.: The author uses EL and IL for *either* epistemic and intuitionistic logic *or* explicit and implicit logic by context.

5 Choice or co-existence: translations and merges
But first it may seem time for a choice. Is intuitionistic logic or epistemic logic better or deeper as an analysis of information and knowledge? Should we prefer one over the other? Many philosophers think in this style, but we feel that this adversarial attitude is not very productive, and it also runs counter to known mathematical facts about system connections …

Already in Gödel's seminal .. , there is a faithful translation from intuitionistic logic into the modal logic S4 whose underlying intuition follows the present knowledge perspective. We now look at this connection to see what it achieves.

Translating IL Into EL The *Gödel translation t* turns the intuitionistic truth definition into a syntactic recipe, according to the following recursive clauses: …

t(¬ϕ) = □¬t(ϕ) (5.4.1)

LET p, q, r, s: ϕ,q, *t*, ψ.

(r&~p)=#(~r&p) ; TCTC **F**T**F**T TCTC **F**T**F**T (5.4.2)

t(ϕ → ψ) = □(t(ϕ) → t(ψ)) (5.5.1)

(r&(p>s))=#((r&p)>(r&s)) ; `CCCC NTNT CCCC NNNN` (5.5.2)

Remark 5.4/.5: Eqs. 5.4.2 and 5.5.2 as rendered are *not* tautologous, refuting the recursive translation of "the intuitionistic truth definition into a syntactic recipe".

… This explains key features of intuitionistic logic in modal terms. For instance, varieties of implication place different demands on knowledge:

intuitionistic $\phi \rightarrow \psi$ is $\Box(\phi \rightarrow \psi)$, (5.6.1)

(p>s)=#(p>s) ; `NTNN NTNN NTNN NTNN` (5.6.2)

the earlier $\neg\phi \vee \psi$ is the stronger $\Box\neg\phi \vee \psi$, (5.7.1)

(~p+s)=(#~p+s) ; `NTNT NTNT TTTT TTTT` (5.7.2)

and $\neg(\phi \wedge \neg\psi)$ the weaker $\Box(\phi \rightarrow \psi)$. (5.8.1)

~(p&~s)=#(p>%s) ; `NTNT NTNT NNNN NNNN` (5.8.2)

Remark 5.6/.8: Eqs. 5.6.2-5.8.2 are *not* tautologous, refuting "key features of intuitionistic logic in modal terms".

6 Dynamic Logic of Information Change
Public Announcement Logic
Public announcements are studied in PAL, a system that extends epistemic logic with a dynamic modality for truthful announcements … . This dynamic modality has a complete logic that can analyze delicate phenomena, such as complex epistemic assertions, say of current ignorance, changing truth value under update. This typically shows in order dependence: a sequence !Kp ; !p makes sense, but !p ; !¬Kp is contradictory. Here we only display the 'recursion law' for knowledge after update, which is the basic dynamic equation of hard information:

$[!\phi]K\psi \leftrightarrow (\phi \rightarrow K(\phi \rightarrow [!\phi]\psi))$ (6.1.1)

LET p, q, r, s: ϕ, $!\phi$, K, ψ.

(q&(r&s))=(p>(r&(p>(q&s)))) ;**F**T**F**T **F**T**F**T **F**T**F**T **F**TTT (6.1.2)

Remark 6.1.2: Eq. 6.1.2 is *not* tautologous, hence refuting "the recursion law after knowledge update [as] the basic dynamic equation of hard information" for public announcement logic.