Blueshift and Redshift In Wide Angle Diffraction

Eric Su
eric.su.mobile@gmail.com
https://sites.google.com/view/physics-news/home
(Dated: July 18, 2019)

The observation of spectral shift in astronomy bears great similarity to the frequency shift in the Doppler effect. Both blueshift and redshift can be described by the movement of the double-slit interference. In the rest frame of the star, the light passes through the slit to travel a straight path to reach the projection screen. The intersection of this path and the screen determines how the spectrum is shifted. If the screen moves away from the path, the spectrum will be shifted away from the center of the screen. This is known as redshift. If the screen moves toward the path, the spectrum will be shifted toward the center of the screen. This is known as blueshift. The spectrum not only shifts in position but also resizes proportionally. The spectral shift is caused by the motion of earth in the rest frame of the star while the wavelength of the star light remains constant. The redshift places a maximum limit on the radial velocity of the remote galaxy. The galaxy can not be detected if the earth moves faster than the light in the rest frame of the galaxy. This is dark galaxy.

I. INTRODUCTION

The interference pattern generated by the diffraction grating often shows a spectral shift corresponding to the movement of the star. The movement is actually the relative motion between the star and the earth. Therefore, the same spectral shift can be observed in the rest frame of the star as well as in the rest frame of the earth.

In the rest frame of the star, the diffraction grating moves toward or away from the star. The wavelength is not affected by the motion of the grating. The light travels from the slit on the grating to the projection screen (or CCD) to create a path of phase shift. The path is stationary relative to the star.

In the rest frame of the earth, the path of phase shift moves together with the star. It undergoes parallel transport along the radial direction toward the star. If the path is transported toward the screen, the intersection of the path and the screen will be closer to the center of the screen. The interference pattern shifts toward the center of the screen. If the path is transported away from the screen, the intersection of the path and the screen will move away from the center of the screen. The interference pattern shifts away from the center of the screen.

The resulting spectral shift is called either blueshift or redshift.

II. PROOF

A. Double-Slit Interference

A light emitter emits coherent light along the x-direction through a plate with two parallel slits to reach a projection screen. Both the plate and the screen are aligned with the y-z plane. The emitter, the plate, and the screen are all stationary relative to a reference frame $F_1$.

A series of alternating light and dark bands appear on the projection screen along the y-direction. Let the distance between the plate and the screen be $D_0$. The displacement of the light band from the center of the screen is $y_0$. The separation between the parallel slits is $d_0$.

If $d_0 << y_0$ and $y_0 << D_0$, the constructive interference can be described by the equation of phase shift\[1\] for the constructive phase difference as

$$y_0 = m \cdot \lambda_1 \cdot \frac{\sqrt{D_0^2 + y_0^2}}{d_0}$$

(1)

$\lambda_1$ is the wavelength in $F_1$. $m$ is a positive integer. The derivation of equation (1) is located next to the conclusion.

B. Stationary Star

Let both the plate and the screen move at a constant velocity $(v,0,0)$ relative to $F_1$. The rest frame of both the
plate and the screen becomes $F_2$. The interference pattern is shifted as the result of the relative motion between $F_1$ and $F_2$. The equation of phase shift becomes

$$y_1 = m \ast \lambda_1 \ast \frac{\sqrt{D_1^2 + y_1^2}}{d_1}$$

(3)

The choice of inertial reference frame along the x-direction has no effect on the measurement along the y-direction.

$$d_1 = d_0$$

(4)

The length is conserved in all inertial reference frames[6,9]. Let the elapsed time for the light to travel from the slit plate to the projection screen be $T$. The screen has moved a distance of $v \ast T$ by the time the light reaches the screen. The total distance in the x-direction for the light to travel from the plate to the screen is

$$D_1 = D_0 + v \ast T$$

(5)

From equations (1,3,4,5),

$$y_1 = m \ast \lambda_1 \ast \frac{\sqrt{(D_0 + v \ast T)^2 + y_1^2}}{d_0}$$

(6)

The displacement of light band shifts from $y_0$ to $y_1$ due to the relative motion between $F_1$ and $F_2$. The angle formed by the x-axis and the path of phase shift is $\theta_1$.

$$\tan(\theta_1) = \frac{y_1}{D_1}$$

(7)

From equations (2,3,4,7),

$$\theta_1 = \theta_0$$

(8)

The direction of the path of phase shift is independent of the motion of the screen.

C. Stationary Earth

The spectral shift is conserved in all inertial reference frames. In the rest frame of the earth, $F_2$, the motion of star results in the same spectral shift as

$$y_2 = m \ast \lambda_2 \ast \frac{\sqrt{D_2^2 + y_2^2}}{d_2}$$

(9)

The choice of inertial reference frame along the x-direction has no effect on the measurement along the y-direction.

$$y_2 = y_1$$

(10)

$$d_2 = d_1$$

(11)

The wavelength is conserved in all inertial reference frames[3-7].

$$\lambda_2 = \lambda_1$$

(12)

The elapsed time is conserved in all inertial reference frames[8-12]. From equations (4,6,9,10,11,12),

$$D_2 = D_0 + v \ast T$$

(13)

The angle formed by the x-axis and the path of phase shift is $\theta_2$.

$$\tan(\theta_2) = \frac{y_2}{D_2}$$

(14)

From equations (5,7,10,13,14),

$$\theta_2 = \theta_1$$

(15)

The direction of the path of phase shift is conserved in both $F_1$ and $F_2$.

D. Redshift

In the rest frame of the earth, $F_2$, a star is observed to move away from the earth if

$$v > 0$$

(16)

The path of phase shift moves together with the star in $F_1$. This causes the intersection of the path and the screen to move away from the center of the screen.

From equations (2,13,14),

$$y_2 - y_0 = \tan(\theta_2)D_2 - \tan(\theta_0)D_0 = \tan(\theta_0)vT$$

(17)

The spectral shift ratio is

$$\frac{y_2 - y_0}{y_0} = \frac{vT}{D_0} > 0$$

(18)

The interference pattern is shifted away from the center of the screen. This is commonly known as redshift in the astronomy.

E. Blueshift

In the rest frame of the earth, $F_2$, a star is observed to move toward the earth if

$$v < 0$$

(19)

The path of phase shift moves together with the star in $F_1$. This causes the intersection of the path and the screen to move toward the center of the screen.

From equations (2,13,14),

$$y_2 - y_0 = \tan(\theta_2)D_2 - \tan(\theta_0)D_0 = \tan(\theta_0)vT$$

(20)

The spectral shift ratio is

$$\frac{y_2 - y_0}{y_0} = \frac{vT}{D_0} < 0$$

(21)

The interference pattern is shifted toward the center of the screen. This is commonly known as blueshift in the astronomy.
F. Radial Velocity

Let the speed of light be $C$ in $F_1$. From equation (7), the distance for the light to travel from the slit to the screen is

$$C * T = \frac{D_1}{\cos(\theta_1)} = \frac{D_0 + v * T}{\cos(\theta_0)}$$

(22)

$$T * \cos(\theta_0)C = D_0 + v * T$$

(23)

$$T = \frac{D_0}{\cos(\theta_0)C - v}$$

(24)

Let $z$ be the spectral shift ratio.

$$z = \frac{y_2 - y_0}{y_0}$$

(25)

From equations (18, 25),

$$z = \frac{vT}{D_0}$$

(26)

From equations (24, 26),

$$z = \frac{v}{\cos(\theta_0)C - v}$$

(27)

$$v = \frac{z}{1 + z} \cos(\theta_0)C$$

(28)

For $0 < z << 1$,

$$v = z * \cos(\theta_0)C$$

(29)

For $z >> 1$,

$$v = \cos(\theta_0)C$$

(30)

G. Spectral Shift Ratio

The spectral shift ratio, $z$, is related to the popular definition of redshift, $z'$.

$$z' = \frac{\lambda_{obs}}{\lambda_1} - 1$$

(31)

From equation (1), the original spectrum is represented by

$$y_0 = m * \lambda_1 * \sqrt{\frac{D_0^2 + y_0^2}{d_0}}$$

(32)

Modern astronomy assumes that the wavelength changes from $\lambda_1$ to $\lambda_{obs}$ but both $D_0$ and $d_0$ remain constant. The shifted spectrum is represented by

$$y_2 = m * \lambda_{obs} * \sqrt{\frac{D_0^2 + y_2^2}{d_0}}$$

(33)

From equations (25, 32, 33),

$$z = \frac{y_2 - y_0}{y_0} = \frac{\lambda_{obs}}{\lambda_1} \sqrt{\frac{D_0^2 + y_0^2}{d_0^2 + y_0^2}} - 1$$

(34)

From equations (31, 34),

$$z = \frac{y_2 - y_0}{y_0} = (z' + 1) \sqrt{\frac{D_0^2 + y_0^2}{d_0^2 + y_0^2}} - 1$$

(35)

From equations (2, 35),

$$z = (z' + 1) \cos(\theta_0)/\cos(\theta_{obs}) - 1$$

(36)

It is clearly that $z'$ is not an accurate value for $z$ if $\cos(\theta_{obs})$ decreases significantly. $z'$ can not represent the spectrum accurately. This problem arises from the assumption that the wavelength is not constant. Without any verification, modern astronomy believes the wavelength has changed from $\lambda_1$ to $\lambda_{obs}$.

The first table shows the predicted radial velocity for large $z$.

**TABLE I. Radial Velocity and Comparison of $z$ and $z'$**

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>$z$</th>
<th>$z'$</th>
<th>Radial Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>GN-z11[13,14]</td>
<td>11.18</td>
<td>11.148</td>
<td>0.9179C</td>
</tr>
<tr>
<td>GN-108036[15]</td>
<td>9.27</td>
<td>7.247</td>
<td>0.9026C</td>
</tr>
<tr>
<td>ULAS J1120+0641[16]</td>
<td>7.086</td>
<td>7.082</td>
<td>0.8763C</td>
</tr>
<tr>
<td>IOK-1[17,18]</td>
<td>6.905</td>
<td>6.603</td>
<td>0.8735C</td>
</tr>
</tbody>
</table>

The second table shows the first order angle related to the wavelength. From equation (32),

$$\sin(\theta_0) = \frac{\lambda_1}{d_0}$$

(37)

From equation (33),

$$\sin(\theta_{obs}) = \frac{\lambda_{obs}}{d_0}$$

(38)

**TABLE II. Wavelength and Angle**

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>Grating</th>
<th>$\lambda_1$</th>
<th>$\sin(\theta_0)$</th>
<th>$\lambda_{obs}$</th>
<th>$\sin(\theta_{obs})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GN-z11</td>
<td>30.8</td>
<td>0.00372</td>
<td>1470</td>
<td>0.0452</td>
<td></td>
</tr>
<tr>
<td>GN-108036</td>
<td>600.0</td>
<td>0.0726</td>
<td>998</td>
<td>0.5988</td>
<td></td>
</tr>
<tr>
<td>ULAS J1120+0641</td>
<td>32</td>
<td>0.00387</td>
<td>978</td>
<td>0.0313</td>
<td></td>
</tr>
<tr>
<td>IOK-1</td>
<td>300.0</td>
<td>0.0363</td>
<td>920</td>
<td>0.276</td>
<td></td>
</tr>
</tbody>
</table>

H. Dark Galaxy

In the rest frame of GN-z11, the earth is moving away at the speed of 0.9179C. The light from GN-z11 slowly
catches up with the earth. The speed difference between the light and the earth is only 0.082C which creates a spectral shift known as redshift in the rest frame of the earth.

From equation (28), the maximum speed of the earth in the rest frame of the galaxy is less than \( \cos(\theta_0)C \).

For greater \( v \), the speed difference between the light and the earth decreases to negative. The earth moves faster than the light in the rest frame of the remote galaxy. The light can not reach the earth. Such galaxy is invisible to the earth. No electromagnetic radiation from it can reach the earth. Therefore, the maximum radial velocity from a redshift spectrum can not exceed \( \cos(\theta_0)C \).

I. Experimental Verification

The spectra may vary greatly at different radial velocities at first glance. In fact, the spectra of the same galaxy are proportional to each other.

From equations (8,15),

\[ \theta_2 = \theta_0 \]

(39)

The spectrum not only shifts in position but actually resizes proportionally.

![FIG. 2. A galaxy spectrum at four different redshifts](image)

J. Doppler Effect

Let the observed frequency of the light from the distant star be \( f_1 \) in \( F_1 \) but \( f_2 \) in \( F_2 \). According to the Doppler effect[2], the frequency decreases if the star is moving away from the earth.

\[ f_2 < f_1 \]

(40)

The frequency increases if the star is moving toward the earth.

\[ f_2 > f_1 \]

(41)

The speed of the star light in \( F_1 \) is

\[ c_1 = f_1 \times \lambda_1 \]

(42)

The speed of the star light in \( F_2 \) is

\[ c_2 = f_2 \times \lambda_2 \]

(43)

From equations (12,40,42,43), the speed of star light decreases if redshift is observed.

\[ c_2 < c_1 \]

(44)

From equations (12,41,42,43), the speed of star light increases if blueshift is observed.

\[ c_2 > c_1 \]

(45)

K. Derivation of Equation of Phase Shift

The distance from the upper slit to the intersection point is

\[ L_+ = \sqrt{D_0^2 + (y_0 - \frac{d_0}{2})^2} \]

(46)

The distance from the lower slit to the intersection point is

\[ L_- = \sqrt{D_0^2 + (y_0 + \frac{d_0}{2})^2} \]

(47)

The condition for a constructive interference is

\[ L_+ - L_- = m\lambda_1 \]

(48)

From equations (46,47),

\[ L_+^2 - L_-^2 = 2d_0 \times y_0 \]

(49)

\[ L_+ - L_- = \frac{2d_0 \times y_0}{L_+ + L_-} \]

(50)

If \( d_0 << y_0 \),

\[ L_+ + L_- = 2\sqrt{D_0^2 + y_0^2} \]

(51)

From equations (50,51),

\[ L_+ - L_- = \frac{d_0 \times y_0}{\sqrt{D_0^2 + y_0^2}} \]

(52)

From equations (48,52),

\[ y_0 = m \times \lambda_1 \frac{\sqrt{D_0^2 + y_0^2}}{d_0} \]

(53)
III. CONCLUSION

The spectral shift is equivalent to the frequency shift in the Doppler effect. Both the spectrum and the frequency transform according to the motion of the observer and the motion of the light source. As the frequency of the light changes due to the motion of the observer, so does the spectrum of the light due to the motion of the earth.

The spectrum is made of the intersection point between the path of phase shift and the projection screen (CCD).

The intersection moves closer to the center of the screen if the screen moves closer to the path. The blueshift is observed.

The intersection moves away from the center of the screen if the screen moves away from the path. The redshift is observed.

The shifting of a spectrum is similar to the zooming of a digital photo. By zooming into the spectrum, the redshift is observed. By zooming out of the spectrum, the blueshift is observed. Modern astronomy never realizes that the shifted spectrum is actually the original spectrum but of different size.

The spectral shift ratio is unique to the movement of the star. For large ratio exceeding 1, the radial velocity approaches the speed of light. It can not exceed the speed of light because such galaxy can not be detected with light if it moves faster than the light. In the rest frame of the dark galaxy, the earth moves faster than the star light. No electromagnetic radiation can reach the earth. The galaxy is called dark galaxy.

In the rest frame of the star, the wavelength of its light remains constant while the motion of the earth causes the spectrum to transform accordingly.

In fact, the wavelength is conserved in all reference frames[3-7]. The relative motion of the star transforms the frequency of its light but not the wavelength. The apparent frequency of the star light is different from the original frequency. The apparent wavelength is identical to the original wavelength.

[1] "Double Slit Interference", http://www.schoolphysics.co.uk/age16-19/Wave