A New Axiom in Set Theory

Abstract. We define a new axiom in set theory. Any set all of whose elements can be named is a denumerable set. This sets up 1-1 correspondence between the real numbers in the closed interval \([0, 1]\) and the natural numbers.

The positive integers can be put in one to one correspondence with the terminating decimal fractions in the open interval \((0, 1)\). \(1 \rightarrow .1, \ 2 \rightarrow .2, \ldots, \ 10 \rightarrow .01, \ldots\) Each terminating decimal is the mirror image reflection through the decimal point of a positive integer. The mapping does not include any repeating decimal fractions. From this mapping the set of all rational numbers would appear to be uncountable. This shows that attempting to map the real numbers in the closed interval \([0, 1]\) to the natural numbers, by listing them as unending decimal fractions is futile. Cantor’s diagonal proof that the real numbers are an uncountable set can never even get started.

The problem is in defining the real numbers in \([0, 1]\) as the set of all unending decimal expansions. That’s vague. Most non-algebraic real numbers cannot be explicitly referenced. We need a new definition.

Each real number in the closed interval \([0, 1]\) is:

\[
S = \lim_{m \to \infty} \sum_{n=1}^{m} a_n/10^n
\]

\(S\) is the limit of all the partial decimal sums.

For the terminating decimals in \([0, 1]\) the subscript of \(S\) is the negative of the natural number that is reflected through the decimal point.

\[.1 \rightarrow S_{-1} \quad .2 \rightarrow S_{-2} \quad .01 \rightarrow S_{-10} \] for zero \(.0 \rightarrow S_0\).

We define a new axiom in set theory. Any set all of whose elements can be named is a denumerable set. Any named element in a set can be counted.

Every unending decimal in \([0, 1]\) is the limit of its partial decimal sums.

To represent them we make an infinite grid leaving out any duplicates.

The first column is all the potential unending decimals represented by \(.1\ldots\)

The second column is all the potential unending decimals represented by \(.2\ldots\)

The third column is all the potential unending decimals represented by \(.3\ldots\) and so forth.

We use ordered pairs of positive integers to represent the numbers in the grid.

\(T(a, b)\) represents the limit of \(a(th)\) unending decimal represented by \(.b\ldots\)

For all positive integers \(n\) we make a diagonal mapping of \(S_n\) to \(T(a, b)\).

\[S_1 \rightarrow T(1, 1) \quad S_2 \rightarrow T(1, 2) \quad S_3 \rightarrow T(3, 1) \quad S_4 \rightarrow T(1, 3) \quad S_5 \rightarrow T(2, 2) \ldots\]

All the decimals in \([0, 1]\) can be mapped to the natural numbers. We map the terminating decimals to the odd numbers and the unending decimals to the even numbers.

\[0 \rightarrow S_0, \ 1 \rightarrow S_{-1}, \ 2 \rightarrow S_1, \ 3 \rightarrow S_{-2}, \ 4 \rightarrow S_2, \ 5 \rightarrow S_{-3}, \ 6 \rightarrow S_3, \ldots\]

Explore the detailed proofs and fascinating consequences of the real numbers as a denumerable set in

https://arxiv.org/abs/1002.4433

Addressing mathematical inconsistency: Cantor and Gödel refuted by J. A. Perez.

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