Refutation of interpretability logics

Abstract: Of the non trivial logics for axioms as evaluated, none is tautologous. Hence the interpretability logic **IL** is refuted, and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠, ⊳ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ▽, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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Abstract We obtain modal completeness of the interpretability logics $\mathbf{ILP_0}$ and **ILR** w.r.t. generalized Veltman semantics.

1 Introduction

1.1 Interpretability logics

The language of interpretability logics is given by A ::= p | ⊥ | A → A | A A, where p ranges over a countable set of propositional variables. Other Boolean connectives are defined as abbreviations, as usual. Since A can be defined (over extensions of IL) as an abbreviation too (expanded to ¬A ⊥), we do not include □ or ◊ in the language. If A is constructed in this way, we will say that A is a modal formula.

Definition 1. Interpretability logic **IL** is given by the following list of axiom schemata.

1. classical tautologies (in the new language);

K. □(A → B) → (□A → □B); [trivial tautology]

L. □(□A → A) → □A; (1.2.1)

LET p, q, r: A, B, C.

#(#p>p)>#p ; CTCT CTCT CTCT CTCT (1.2.2)

J1. □(A → B) → A ⊳ B; [trivial tautology]

J2. (A ⊳ B) ∧ (B ⊳ C) → A ⊳ C; [trivial tautology]

J3. $(A \rhd C) \wedge (B \rhd C) \to A \vee B \rhd C$; (1.5.1)

((p>r)&(q<r))>(p+(q>r)) ; `TT`**`F`**`T TTTT TT`**`F`**`T TTTT` (1.5.2)

J4. $A \rhd B \to (\Diamond A \to \Diamond B)$; [trivial tautology]

J5. $\Diamond A \rhd A$. (1.7.1)

%p>p ; `NTNT NTNT NTNT NTNT` (1.7.2)

Of the non trivial logics for axioms as evaluated, Eqs. 1.2.2, 1.5.2, and 1.7.2, none is tautologous. Hence the interpretability logic **IL** is refuted.